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Some Applications of Simplex Method

Divya K. Nadar

Research Scholar, Instrumentation and Control Department, AITS, Rajkot, Gujarat, India

ABSTRACT

In this review paper, simplex method and its application are discussed. The simplex method has played a vital role during these many years in many real-world problems and is still improving in order to get the optimum solution. There are many applications but a total of three applications are taken, which describes the implementation of the simplex method and its computational advantages with examples. It is one of the popular algorithms for linear programming (Maros, 2012. Computational techniques of the simplex method, vol. 61. Springer Science & Business Media).

Keywords: Primal–dual, Simplex method, Model predictive control, Dantzig's method

INTRODUCTION

Linear programming (LP) is an optimisation method that is applicable for the solution of problems in which the objective function and the constraints appear as linear functions of the decision variables. Most real-life problems when formulated as an LP model have more than two variables and therefore need a more efficient method suggesting an optimal solution for such problems. George B. Dantzig, who was a member of the Air Force group, formulated the general LP problem and devised the simplex method of solution in 1947 [3,4]. This has become a significant step in bringing LP into wider use. The concept of simplex LP is considered as a revolutionary development that permits us to make optimal decisions in complex situations. Simplex method is an iterative process but progressively approaches and eventually reaches the optimum point[2].

The number of applications of LP has been very large. One of the early industrial applications of LP was made in the petroleum refineries. In general, an oil refinery has a choice of buying crude oil from several different sources with differing compositions and at differing prices. It can manufacture different products, such as aviation fuel, diesel fuel and gasoline, in varying quantities. The constraints may be due to the restrictions on the quantity of the crude oil available from a particular source, the capacity of the refinery to produce a particular product and so on. A mix of the purchased crude oil and manufactured products is sought that gives the maximum profit. The optimal production plan in manufacturing firm can also be decided using the LP. As the sales of a firm fluctuate, the company can have various options. It can build up an inventory of manufactured products to carry it through the period of peak sales, but this involves an inventory holding cost. It can also pay overtime rates to achieve higher production during periods of higher demand.

Although several other methods have been developed over the years for solving LP problems, the simplex method continues to be the most efficient and popular method for solving general LP problems. In this paper, review and formulation of applications of the simplex method are discussed.

REVIEW OF APPLICATIONS

Case 1

In the first case, optimisation of sand casting is done by using the Dantzig's simplex method. This method is used to explore optimisation of the sand-casting parameters for the most favourable conditions. Aluminium alloys were cast and undergoes a series of mechanical tests. Some process constraints and linear functions are formulated and utilise the simplex method for optimisation of parameters, and the results are used for studying performance of the parameters. Variation of casting properties is shown in Table 1 [7].

Table 1: Variation of casting properties with mould temperature

Pouring Temp. (°C)	Solidification Time (min)	Impact Strength (J/mm^2)	UTS (N/mm^2)	Hardness (HRB)	Percentage Elongation (%)
700	1.20	0.46	44.20	15.08	1.80
750	1.47	0.32	50.50	16.47	2.50
800	2.32	0.31	64.20	17.60	6.80
850	3.24	0.30	67.40	18.22	9.90

Case 2

In the second case, primal–dual simplex algorithm is used for solving LP with trapezoidal fuzzy numbers. In this case, two methods are used for solving fuzzy linear programming (FLP) problems using trapezoidal numbers without turning them into a crisp LP. This method is proposed by Ganesan and Veeramani[4] and the dual-simplex is given by Ebrahimnejad and Nasseri[5]. The primal method alone cannot solve the problems as it has some loopholes in it and the dual method doesn't give the feasible solution, so both the methods are combined into a primal–dual algorithm that is applied to overcome the loopholes.

Case 3

In the third case, a problem is minimised in model predictive control (MPC) by using a modified simplex method. In this case, minimisation is done and method is applied to a polyhedron and is changed according to the problem, thus making the solution feasible and faster. The method skips iterations and its advantages are verified in the examples.

FORMULATION OF APPLICATION

Case1

The simplex method solves the problem by converting inequalities into a LP and then solves by manipulation as this method is efficient and easy to implement in a problem. A number of different methods are used to solve the problems for the optimal result so the following subsequent paragraphs would make this method more clear regarding which is better compared to the other methods. There are five casting parameters (solidification time, impact strength, ultimate tensile strength, hardness and percentage elongation) on the three variables (mound temperature, pouring temperature and runner size) which are required for preparing, casting and moulding, and graphs are also required in which deviation of different process parameters are measured, which can be positive or negative, and thus finally one equation with three constraints are derived from the graph, and table and the function is minimised by applying simplex method, and the optimal solution is 0.392 (solidification time) which is feasible according to the problem.

Case 2

In the second case, the coefficients involved in the objective and constraint functions are not precise in nature and have to be interpreted as fuzzy numbers to reflect the real-world situation. Thus, the mathematical problem is, therefore, referred to as a fuzzy mathematical programming problem. Maleki *et al.* proposed a simple method for solving fuzzy number linear programming (FNLP) problems. They also applied a special kind of FNLP problems, involving fuzzy numbers only in objective function, as an auxiliary problem. A certain linear ranking function is used to define the dual of FNLP problem as a similar problem that leads to an efficient algorithm, called the dual simplex algorithm for solving FNLP problems. The concept of the symmetric triangular fuzzy number and an approach to defuzzify a general fuzzy quantity is applied in this algorithm. The new method, based on the primal simplex algorithm, is for solving LP problem with symmetric trapezoidal fuzzy numbers without converting them to crisp LP problems. Some concepts of fuzzy set are explained in this case and fuzzy primal simplex algorithm and also fuzzy primal–dual algorithm which start with a basic feasible solution for FLP, but the solution is not feasible and so a new dual-simplex algorithm has been developed to overcome the shortcoming by the duality. This algorithm starts with dual basic feasible solution and walks to an optimal solution by moving along the adjacent dual basic feasible solution. As a result, a fuzzified version of conventional primal–dual method is developed. An illustrative example is taken, in which slack and fuzzy artificial variables are introduced to minimise the function and gives the basic feasible solution. This process is time consuming and has no efficiency.

Case 3

Finally, in the third case, simplex method is used for minimisation of a problem and it's formulation on the basis of some manners:

A new variable is introduced for minimising the sum.

By replacing each deviation by a difference of two non-negative variables and then minimising it.

A modified simplex method has the following three objectives:

1. Simplex method is used to find the optimal point in a polyhedron that extends into the negative domain.
2. To skip the unnecessary steps thus avoiding any computational effort.
3. To formulate the problem as its initial basic feasible solution is available.

This method is presented for 1-norm minimisation problem as it arises in MPC. The computational advantages of the method are verified by its application by using it in examples.

Example 1: To enable the simplex method to find the optimal point in a polyhedron that extends into the negative domain.

The basic strategy is to use the simplex method in the usual manner on the section (portion) of the polyhedron that lies in the positive domain to locate the optimal point in this section. Then, if the search needs to go into the negative domain, flip the section of the polyhedron that is of interest into the positive domain and continue the search. The flipping can be accomplished by switching the sign of an optimisation variable through redefinition. The slack variables are not allowed to switch. A record is kept of the switch status of the optimisation variables to report their correct sign at the end of the solution. This is shown graphically in the paper.

Example 2: A open loop transfer function is taken and the results are computed by using a computer program as there would be number of iterations and many parameters are compared by using different algorithms, and, thus, this method uses a smaller number of steps, and most modern control packages use this method to solve the optimisation problem, and many control system can be benefitted by using the simplex method.

CONCLUSION

In case one, the below conclusions can be drawn, the Dantzig's simplex method can be adapted to casting process to explore optimisation of some sand-casting parameters for the best results. As researched with simplex method, the minimum deviation (ΔS) of the solidification time was obtained as 0.392, this is the minimum deviation from the ideal value that can be tolerated to achieve optimal combination of other factors to obtain a good product. Although a decrease in solidification time results in finer microstructure leading to improvement in the entire range of mechanical properties, the result from the model would practically imply that experimental solidification time can be further reduced, whereas still obtaining a balance of other casting factors[7]. Thus in the second case, a new approach based on primal simplex algorithm to obtain the solution of FLP with trapezoidal fuzzy numbers without converting them into crisp linear problem and is based on the interesting results given by Ganesan and Veeramani[3]. This method is expected to be reliable for solving minimum cost flow with fuzzy parameters in which finding an initial dual feasible solution turns out to be a trivial task[8]. In the third case, thus by using the simplex method, we can find the optimal point in a polyhedron that extends into the negative domain. Thus the need to increase the size of the LP problem to bring the entire polyhedron into the positive domain is eliminated. The unnecessary iterations in the search can be skipped. The modifications presented can also be used in problems other than the 1-norm minimisation problem. The modified simplex method offers a significant computational advantage over the conventional formulations for the solution of 1-norm minimisation problems[9].

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