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RESEARCH ARTICLE

Controllability for the Nonlinear Fuzzy Neutral Integrodifferential Equations with Nonlocal Conditions

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ABSTRACT:
In this paper, we devoted study the controllability for the nonlinear fuzzy neutral integrodifferential equations control system in EN. Moreover we study the fuzzy solution for the normal, convex, upper semicontinuous, and compactly supported interval fuzzy number. The results are obtained by using the Banach Fixed point theorem.

KEY WORDS: Fuzzy set, fuzzy number, neutral integrodifferential system, fuzzy solution, fixed point theorem.

1. INTRODUCTION:

The term ‘‘fuzzy differential equation’’ was coined in 1978 by Kandel and Byatt (1978). There are many suggestions to define a fuzzy derivative. One of the earliest was to generalize the Hukuhara derivative of a set-valued function. This generalization was made by Puri and Ralescu (1983) and studied by Kaleva (1987). It soon appeared that the solution of fuzzy differential equation interpreted by Hukuhara derivative has a drawback: it became fuzzier as time goes. Hence, the fuzzy solution behaves quite differently from the crisp solution. To alleviate the situation, Hüllerme-ier (1997) interpreted fuzzy differential equation as a family of differential inclusions. The main shortcoming of using differential inclusions is that we do not have a derivative of a fuzzy number-valued function. There is another approach to solve fuzzy differen-tial equations which is known as Zadeh’s extension principle (Misukoshi, ChalcoCano, Román-Flores, and Bassanezi, 2007; Oberguggenberger and Pittschmann, 1999), the basic idea of the extension principle is: consider fuzzy differential equation as a deterministic differential equation then solve the deterministic differential equation. After getting deterministic solution, the fuzzy solution can be obtained by applying extension principle to deterministic solution. But in Zadeh’s extension principle we do not have a derivative of a fuzzy number-valued function either. In Bede and Gal (2005) and Bede, Rudas, and Bencsik (2007), strongly generalized derivative concept was introduced. This concept allows us to solve the mentioned shortcomings and in Khastan, Bahrami, and Ivaz (2009) authors studied higher order fuzzy differential equations with strongly generalized derivative concept.

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Recently, Gasilov, Amrahov, and Fatullayev (2011) proposed a new method to solve a fuzzy initial value problem for the fuzzy linear system of differential equations based on properties of linear transformations. But they used fuzzy bunch of functions instead of fuzzy number valued functions. In recent paper, Y. C Kuwun, J. S Hwang, J.S Park and J. H Park, Controllability for the Impulsive Semilinear Fuzzy Integrodifferential equations with nonlocal conditions can be extended to the fuzzy neutral integrodifferential equations. We establish a synthesis of crisp solution of fuzzy initial value problem and the method proposed in Kaleva (1987) to solve fuzzy initial value problem. To do this firstly we remained the following basic concepts from fuzzy arithmetic and fuzzy calculus.

2. PRELIMINARIES

In section, we shall introduce some basic definitions, notations, lemmas and result which are used throughout this paper. A fuzzy subset of $\mathbb{R}^n$ is defined in terms of a membership function which assigns to each point $x \in \mathbb{R}^n$ a grade of membership in the fuzzy set. Such a membership function is denoted by $u : \mathbb{R}^n \to [0,1]$.

Throughout this paper, we assume that u maps $\mathbb{R}^n$ onto $[0,1]$, $[u]^0$ is a bounded subset of $\mathbb{R}^n$, u is upper semicontinuous, and u is fuzzy convex. We denote by E^n the space of all fuzzy subsets u of $\mathbb{R}^n$ which are normal, fuzzy convex, and upper semicontinuous fuzzy sets with bounded supports. In particular, E^1 denotes the space of all fuzzy subsets u of $\mathbb{R}$. A fuzzy number a in real line $\mathbb{R}$ is a fuzzy set characterized by a membership function χ_a

$\chi_a : \mathbb{R} \to [0,1]$. A fuzzy number a is expressed as $a = \int_{x \in \mathbb{R}} \frac{\chi_a}{x}$ with the understanding that $\chi_a(x) \in [0,1]$, represents the grade of membership of x in a and $\int$ denotes the union of $\frac{\chi_a}{x}$.

Definition 2.1 *A fuzzy number $a \in \mathbb{R}$ is said to be convex if, for any real numbers x, y, z in $\mathbb{R}$ with $x \le y \le z$, $\chi_a(y) \ge \min\{\chi_a(x), \chi_a(z)\}$.*

Definition 2.2 *The height of a fuzzy set is the largest membership value attained by any point.*

Definition 2.3 *If the height of a fuzzy set equals one, then the fuzzy set is called normal. Thus, a fuzzy number $a \in \mathbb{R}$ is called normal, if the followings holds: $\max_x \chi_a(x) = 1$.*

Result 2.1 *Let E_N be the set of all upper semicontinuous convex normal fuzzy numbers with bounded α - level intervals (see [33]). This means that if $a \in E_N$, then $\alpha -$ level set* $[a]^\alpha = \{x \in \mathbb{R} : a(x) \ge \alpha, 0 \le \alpha \le 1\}$, is a closed bounded interval, which we denote by $[a]^\alpha = [a_q^\alpha, a_r^\alpha]$ and there exists a $t_0 \in \mathbb{R}$ such that $a(t_0) = 1$.

Result 2.2 *Two fuzzy numbers a and b are called equal $a = b$, if $\chi_a(x) = \chi_b(x)$, for all $x \in \mathbb{R}$. It follows that* $a = b \Leftrightarrow [a]^\alpha = [b]^\alpha$, *for all* $\alpha \in (0,1]$.

Result 2.3 *A fuzzy number a may be decomposed into its level sets through the resolution identity*

$a = \int_0^1 \alpha [a]^\alpha$, where $\alpha[a]^\alpha$ is the product of a scalar α with the set $[a]^\alpha$ and $\int$ is the union of $[a]^\alpha$ with α ranging from 0 to 1.

Definition 2.4 *The support of a fuzzy set A in the universal set U is a crisp set that contains all the elements of U that have nonzero membership values in A, that is,* $supp(A) = \{x \in U : \chi_a(x) > 0\}$, where $supp(A)$ denotes the support of fuzzy set A. Hences the support Γ_a of a fuzzy number a is defined, as a special case of level set, by the following: $\Gamma_a = \{x : \chi_a(x) > 0\}$.

Definition 2.5 *A fuzzy number $a \in \mathbb{R}$ is said to be positive if $0 < a_1 < a_2$ holds for the support $\Gamma_a = [a_1, a_2]$ of a, that is, Γ_a is in the positive real line. Similarly, a is called negative if $a_1 \le a_2 < 0$ and zero if*

Let x be a point in $\mathbb{R}^n$ and A be a nonempty subsets of $\mathbb{R}^n$. We define the Hausdroff separation of B from A by $d(x, A) = \inf\{\|x - a\| : a \in A\}$. Now let A and B be nonempty subsets of $\mathbb{R}^n$. We define the Hausdroff separation of B from A by $d_H^*(B, A) = \sup\{d(b, A) : b \in B\}$. In general, $d_H^*(A, B) \ne d_H^*(B, A)$. We define the Hausdroff distance between nonempty subsets of A and B of $\mathbb{R}^n$ by $d_H(A, B) = \max\{d_H^*(A, B), d_H^*(B, A)\}$.

This is now symmetric in A and B. Consequently,

1. $d_H(A, B) \ge 0$ *with* $d_H(A, B) = 0$ if and only if $\overline{A} = \overline{B}$;
2. $d_H(A, B) = d_H(B, A)$;
3. $d_H(A, B) \le d_H(A, C) + d_H(C, B)$; for any nonempty subsets of A, B and C of $\mathbb{R}^n$. The Hausdroff distance is a metric, the Hausdroff metric. The supremum metric d_∞ on E^n is defined by

$$d_\infty(u, v) = \sup\{d_H([u]^\alpha, [v]^\alpha) : \alpha \in (0,1]\},$$
$$\text{for all } u, v \in E^n,$$

and is obviously metric on E^n. The supremum metric H_1 on $C(J, E^n)$ is defined by

$$H_1(x, y) = \sup\{d_\infty(x(t), y(t) : t \in J),$$
$$\text{for all } x, y \in C(J : E^n)\}.$$

We assume the following conditions to prove the existence of solution of the equation (2).

3. Controllability of Fuzzy Solution

The notion of controllability is of great importance in mathematical control theory. Many fundamental problems of control theory such as pole-assignment, stabilizability and optimal control may be solved under the assumption that the system is controllable. It means that it is possible to steer any initial state of the system to any final state in some finite time using an admissible control. During the last few decades, several authors have discussed the existence, uniqueness, and asymptotic behavior of the solution of these systems. Apart from these, the study of controllability and observability properties of a system in control theory is certainly, at present, one of the most active interdisciplinary areas of research. Control theory arises in most modern applications. On the other hand, control theory has remained a discipline where many mathematical ideas and methods have fused to produce a new body of important mathematics. In control theory, one of the most important qualitative aspects of a dynamical system is controllability. As far as the controllability problems associated with finite-dimensional systems modeled by ODEs are concerned, this theory has been extensively studied during the last decades. In the finite-dimensional context, a system is controllable if and only if the algebraic Kalman rank condition is satisfied. According to this property, when a system is controllable for some time, it is controllable for all the time. But this is no longer true in the context of infinite-dimensional systems modeled by PDEs. Consider the fuzzy neutral integrodifferential equations

$$\frac{d}{dt}(x(t)-h(t,x(t))) = A(t)[x(t)+\int_0^t G(t-s)x(s)ds] + f(t,x(t))+u(t), \quad t\in J=[0,b] \tag{1}$$

$$x(0)+g(x)=x_0 \tag{2}$$

where $A(t):J\to E_N$ is fuzzy coefficient, E_N is the fuzzy set of all upper semicontinuous, convex, normal fuzzy numbers with bounded $\alpha-$ level intervals, $f:J\times E_N\to E_N$, $h:J\times E_N\to E_N$, $g:E_N\to E_N$ are all nonlinear functions, $G(t)$ is $n\times n$ continuous matrix such that $\frac{dG(t)x}{dt}$ is continuous for $x\in E_N$ and $t\in J$ with $\|G(t)\|\le k$, $k>0$, $u:J\to E_N$ is control function and the function satisfies the following conditions: H1. The nonlinear function $g:J\times E_N\to E_N$ is a continuous function and satisfies the inequality

$$d_H([g(x(.))]^\alpha,[g(y(.))]^\alpha)\le \delta_g d_H([x(.)]^\alpha,[y(.)]^\alpha)$$

H2. The inhomomogeneous term $f:J\times E_N\to E_N$ is continuous function and satisfies a global Libschitz

$$d_H([f(s,x(s))]^\alpha,[f(s,y(s))]^\alpha)\le \delta_f d_H([x(.)]^\alpha,[y(.)]^\alpha)$$

H3. The nonlinear function $h:J\times E_N\to E_N$ is continuous function and satisfies the global lipschitz condition

$$d_H([h(s,x(.))]^\alpha,[h(s,y(.))]^\alpha)\le \delta_h d_H([x(.)]^\alpha,[y(.)]^\alpha)$$

H4. S(t) is the fuzzy number satisfies for $y\in E_N$, $S'y\in C(J,E_N)\cap C(J,E_N)$ the equation

$$\frac{d}{dt}S(t)y = A(t)(S(t)y+\int_0^t G(t-s)S(s)yds)$$
$$= A(t)S(t)y+\int_0^t S(t-s)A(s)G(s)ds, \quad t\in J$$

such that $[S(t)]^\alpha=[S_q^\alpha,S_r^\alpha]$ and $S_i^\alpha(t)$, $i=q,r$ are continuous. That is, there exists a constant δ_s such that $\|S_i^\alpha(t)\|<\delta_s$. Therefore , in equation (1)-(2) solution of the form

$$x(t)=S(t)[x_0-g(x)-h(0,x_0-g(x))]+h(t,x(t))+\int_0^t S(t-s)A(s)h(s,x(s))ds$$
$$+\int_0^t S(t-s)f(s,x(s))ds+\int_0^t S(t-s)u(s)ds \tag{3}$$

Definition 3.1 *The equation (3) is controllable if there exists* $u(t)$ *such that the fuzzy solution* $x(t)$ *of (3) satisfies* $x(b)=x^1-g(x)$, *that is* $[x(b)]^\alpha=[x^1-g(x)]^\alpha$, *where* x^1 *is a target set.*

We assume that the linear control system with respect to the nonlinear control system (4.1) is nonlocal controllable. Then

$$x(b)=S(b)[x_0-g(x)]+\int_0^b S(b-s)u(s)ds = x^1-g(x)$$

$$[x(b)]^\alpha=[S(b)[x_0-g(x)]+\int_0^b S(b-s)u(s)ds]^\alpha$$

$$= [S_q^\alpha(b)[x_{0q}^\alpha - g_q^\alpha(x)] + \int_0^b S_q^\alpha(b-s)u_q^\alpha(s)ds, S_r^\alpha(b)x_{0r}^\alpha + \int_0^b S_r^\alpha(b-s)u_r^\alpha(s)ds]$$
$$= [(x^1)_q^\alpha - g_q^\alpha(x), (x^1)_r^\alpha - g_0^\alpha(x)] = [x^1 - g(x)]^\alpha$$

Defined the fuzzy mapping $\zeta : P(R) \to E_N$ by

$$\zeta^\alpha(v) = \begin{cases} \int_0^t S(t-s)v(s)ds, & v \subset \overline{\Gamma}_u, \\ 0, & otherwise \end{cases}$$

Then there exists $\zeta_i^\alpha (i = q, r)$ such that

$$\zeta_q^\alpha(v_q) = \int_0^t S_q^\alpha(t-s)v_q(s)ds, \ v_q \in [u_q^\alpha, u^1]$$
$$\zeta_r^\alpha(v_r) = \int_0^t S_r^\alpha(t-s)v_r(s)ds, \ v_r \in [u^1, u_r^\alpha]$$

We assume that ζ_i's are bijective mappings. Hence the α-set of $u(s)$ are

$$[u(s)]^\alpha = [u_q^\alpha(s), u_r^\alpha(s)]$$
$$= [(\tilde{\zeta}_q^{\alpha})^{-1}((x^1)_q^\alpha - g_q^\alpha(x) - S_q^\alpha(b)[(x_0)_q^\alpha - g_q^\alpha - h_q^\alpha(0, x_0 - g(x))]$$
$$- h_q^\alpha(t, x(t)) - \int_0^t S_q^\alpha(t-s)A_q^\alpha(s)h_q^\alpha(t, x(s))ds$$
$$- \int_0^t S_q^\alpha(t-s)f_q^\alpha(t, x(s))ds,$$
$$(\tilde{\zeta}_r^{\alpha})^{-1}((x^1)_r^\alpha - g_r^\alpha(x) - S_r^\alpha(b)[(x_0)_r^\alpha - g_r^\alpha(x) - h_r^\alpha(0, x_0 - g(x))]$$
$$- h_r^\alpha(t, x(t)) - \int_0^t S_r^\alpha(t-s)A_r^\alpha(s)h_r^\alpha(t, x(s))ds - \int_0^t S_r^\alpha(t-s)f_r^\alpha(t, x(s))ds$$

Then substituting this expression into equation (3) yields α - level set of $x(b)$

$$[x(b)]^\alpha = [S_q^\alpha(b)[(x_0)_q^\alpha - g_q^\alpha(x) - h_q^\alpha(0, x_0 - g(x))] + h(s, x(s)) + \int_0^b S_q^\alpha(b-s)A_q^\alpha(s)h_q^\alpha(s, x(s))ds$$
$$+ \int_0^b S_q^\alpha(b-s)f_q^\alpha(s, x(s))ds + \zeta_q^\alpha(\tilde{\zeta}_q^{\alpha})^{-1}((x^1)_q^\alpha - g_q^\alpha(x) - S_q^\alpha(b)[(x_0)_q^\alpha$$
$$- g_q^\alpha(x) - h_q^\alpha(0, x_0 - g(x))] - h_q^\alpha(s, u(s)) - \int_0^b S_q^\alpha(b-s)A_q^\alpha(s)h_q^\alpha(s, x(s))ds$$
$$- \int_0^b S_q^\alpha(b-s)f_q^\alpha(s, x(s))ds, S_r^\alpha(b)[(x_0)_r^\alpha - g_q^\alpha(x) - h_r^\alpha(0, x_0 - g(x))]$$
$$+ h_r^\alpha(s, u(s)) + \int_0^b S_r^\alpha(b-s)A_r^\alpha(s)h_r^\alpha(s, x(s))ds + \int_0^b S_r^\alpha(b-s)f_r^\alpha(s, x(s))ds$$
$$+ \zeta_r^\alpha(\tilde{\zeta}_r^{\alpha})^{-1}((x^1)_r^\alpha - g_r^\alpha(x) - S_r^\alpha(b)[(x_0)_r^\alpha - g_r^\alpha(x) - h_r^\alpha(0, x_0)] - \int_0^b S_r^\alpha(b-s)A_r^\alpha(s)h_r^\alpha(s, x(s))ds$$
$$- \int_0^b S_r^\alpha(b-s)f_r^\alpha(s, x(s))ds = [(x^1)_q^\alpha - g_q^\alpha(x), (x^1)_r^\alpha - g_r^\alpha(x)] = [x^1 - g(x)]^\alpha.$$

We now set

$$\Omega x(t) = S(t)[x_0 - g(x) - h(0, x_0 - g(x))] + h(t, u(t)) + \int_0^t S(t-s)A(s)h(t, x(s))ds$$

$$+\int_0^t S(t-s)f(s,x(s))ds \quad +\int_0^t S(t-s)\tilde{\zeta}^{-1}(x^1-g(x)-S(b)[x_0-g(x)-h(0,x_0-g(x))]$$
$$-h(t,x(t))-\int_0^b S(b-s)A(s)h(s,x(s))ds-\int_0^b S(b-s)f(s,x(s))ds$$

where the fuzzy mapping $\tilde{\zeta}^{-1}$ satisfied above statement. Now notice that $\Omega x(T)=x^1-g(x)$, which means that the control $u(t)$ steers the equation (u) from the origin to x^1 in the time b provided we can obtain a fixed point of the nonlinear operator Ω. Assume that the hypotheses (H_5) The system (4.1) is linear $f\equiv 0$ is nonlocal controllable. (H_6) $(\delta_h+(\delta_s(\delta_f+\delta_k b)+\delta_h)\leq 1$.

Theorem: 4.2 *Suppose that the hypothses $(H_1)-(H_6)$ are satisfied. Then the equation (3) is a nonlocal controllable.*

Proof. We can easily check that Ω is continuous from $C([0,b]:E_N)$ to itself. For $x,y\in C([0,b]:E_N)$,

$$d_H([\Omega x(t)]^\alpha,[\Omega y(t)]^\alpha)=d_H([S(t)[x_0-g(x)-h(0,x_0-g(x))]+h(t,x(t))$$
$$+\int_0^t S(t-s)A(s)h(s,x(s))ds+\int_0^t S(t-s)f(s,x(s))ds$$
$$+\int_0^b S(b-s)\tilde{\zeta}^{-1}(x^1-g(x)-S(b)[x_0-g(x)$$
$$-h(0,x_0-g(x)]-h(t,x(t))-\int_0^b S(b-s)A(s)h(s,x(s))ds$$
$$-\int_0^b S(b-s)f(s,x(s))ds,[S(t)[x_0-g(y)-h(0,x_0-g(y))]$$
$$+h(t,y(t))+\int_0^t S(t-s)A(s)h(s,y(s))ds+\int_0^t S(t-s)f(s,y(s))ds$$
$$+\int_0^b S(b-s)\tilde{\zeta}^{-1}(x^1-S(b-s)[x_0-g(y)-h(0,x_0-g(y))]-h(t,y(t))$$
$$-\int_0^b S(b-s)A(s)h(s,y(s))ds-\int_0^b S(b-s)f(s,y(s))ds)ds]^\alpha)$$
$$\leq(\delta_s(\delta_g+\delta_{h_1}+\delta_h\delta_g)+\delta_h)d_H([x(s)]^\alpha,[y]^\alpha)+(\delta_s(M_A\delta_h+\delta_f)\int_0^t d_H([x(s)]^\alpha,[y(s)]^\alpha)$$
$$+\delta_h d_H([x(s)]^\alpha,[y]^\alpha)+(\delta_s(\delta_h+\delta_f)\int_0^b d_H([x(s)]^\alpha,[y(s)]^\alpha)$$

Let $\kappa_1=\delta_s(\delta_g+\delta_{h_1}+\delta_h\delta_g)+\delta_h$, and $\kappa_2=\delta_s(M_A\delta_h+\delta_f)$ then we have

$$\leq 2\kappa_1 d_H([x(s)]^\alpha,[y]^\alpha)+\kappa_2(\int_0^t d_H([x(s)]^\alpha,[y(s)]^\alpha)+\int_0^b d_H([x(s)]^\alpha,[y(s)]^\alpha))$$

Therefore, $d_\infty(\Omega x(t),\Omega y(t))=\sup_{\alpha\in[0,1]} d_H([\Omega x(t)]^\alpha,[\Omega y(t)]^\alpha)\leq 2\kappa_1 d_\infty([x(s)]^\alpha,[y]^\alpha)$

$$+\kappa_2(\int_0^t d_H([x(s)]^\alpha,[y(s)]^\alpha)+\int_0^b d_infty([x(s)]^\alpha,[y(s)]^\alpha))$$

Hence $H_1(\Omega x(t),\Omega y(t))=\sup_{t\in[0,b]} d_H([\Omega x(t)]^\alpha,[\Omega y(t)]^\alpha)\leq(2\kappa_1+2\kappa_2 b)H_1(x,y)=(2(\kappa_1+\kappa_2 b))H_1(x,y)$

By hypotheses (H_7), we take sufficiently small b, Ω is a contraction mapping. By Banach fixed point theorem (3) has a unique fixed point $x\in C([0,b]:E_N)$.

4. Example

Consider the fuzzy solution of the nonlinear fuzzy neutral integrodifferential equation of the form:

$$\frac{d}{dt}(x(t)-2tx(t)^2)) = 2[x(t)-\int_0^t e^{-(t-s)}x(s)ds]+3tx(t)^2+u(t), \quad t\in J, \tag{4}$$

$$x(0)+\sum_{k=1}^{n}c_k x(t_k) = 0 \in E_N, \tag{5}$$

where x^1 is target set, and the α - level set of fuzzy number **0**,**2** and **3** are

$[0]^\alpha = [\alpha-1, 1-\alpha],\ for\ \alpha\in[0,1]$, $[2]^\alpha = [\alpha+1, 3-\alpha],\ for\ \alpha\in[0,1]$,

$[3]^\alpha = [\alpha+2, 4-\alpha],\ for\ \alpha\in[0,1]$.Let $G(t-s)=e^{-(t-s)}, f(t,u(t))=3tu(t)^2, h(t,u(t))=2tu(t)^2$. Then

α - level set of $g(x)=\sum_{k=1}^{n}c_k x(t_k)$ is $[g(x)]^\alpha = [\sum_{k=1}^{n}c_k x(t_k)]^\alpha = [\sum_{k=1}^{n}c_k x_q^\alpha(t_k), \sum_{k=1}^{n}c_k x_r^\alpha(t_k)]$

Let us take target set, $x^1 = 2$.Then substituting this expression into the integral system with respect to (4)-(5) yields $\alpha-$ level set of $x(b)$.

$$\begin{aligned}
[x(b)]^\alpha = {} & [S_q^\alpha(b)[(\alpha-1)-\sum_{k=1}^{n}c_k x(t_k)]+t(\alpha+1)(x_q^\alpha)^2(t)+\int_q^t S_q^\alpha(t-s)t(\alpha+1)t(x_q^\alpha)^2(t)ds \\
& +\int_0^t S_q^\alpha(t-s)t(\alpha+2)(x_q^\alpha)^2)ds+\int_0^t S_q^\alpha(t-s)t(\alpha+1)(x_q^\alpha)^2(t)ds) \\
& +\int_0^b S_q^\alpha(b-s)(\zeta_q^\alpha)^{-1}((\alpha+1)-t(\alpha+1)(x_q^\alpha)^2(t)-\int_0^b S_q^\alpha(b-s)t(\alpha+1)(x_q^\alpha)^2(t)ds \\
& -\int_0^t S_q^\alpha(t-s)t(\alpha+2)(x_q^\alpha)^2(s)ds-\int_0^t S_q^\alpha(t-s)t(\alpha+1)(x_q^\alpha(s))^2ds)ds) \\
& [S_r^\alpha(b)(1-\alpha)]t(\alpha+1)t(x_r^\alpha)^2(t)+\int_0^b S_r^\alpha(b-s)t(3-\alpha)t(x_r^\alpha)^2(t)ds \\
& +\int_0^t S_r^\alpha(t-s)t(4-\alpha)(x_r^\alpha)^2)ds+\int_0^t S_q^\alpha(t-s)t(3-\alpha)(x_r^\alpha)^2(t)ds) \\
& +\int_0^b S_r^\alpha(b-s)(\zeta_r^\alpha)^{-1}((3-\alpha)-t(\alpha+1)(x_r^\alpha)^2(t)-\int_0^b S_r^\alpha(b-s)t(3-\alpha)(x_r^\alpha)^2(t)ds \\
& -\int_0^b S_r^\alpha(b-s)t(4-\alpha)(x_r^\alpha)^2(s)ds-\int_0^b S_r^\alpha(b-s)t(3-\alpha)(x_r^\alpha(s))^2ds)ds)] \\
& = [(\alpha+1)-\sum_{k=1}^{n}c_k x_q^\alpha(t_k), (3-\alpha)-\sum_{k=1}^{n}c_k x_r^\alpha(t_k)] = [2-\sum_{k=1}^{n}c_k x(t_k)]^\alpha = [x^1-g(x)]
\end{aligned}$$

Then all condition stated in theorem1 are satisfied, so the system (4)-(5) is controllable on $[0,b]$.

5. CONCLUSION:

In this paper, by using the concept of fuzzy number in E_N, we study the controllability for the nonlinear impulsive fuzzy neutral integrodifferential controll system in E_N and find the sufficient conditions of controllability for the controll system (1)-(2).

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