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Research Note

Iterative Solutions for Alternate Depths in Open Channels

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ABSTRACT

Computation of alternate depths (super-critical and sub-critical) for a specific energy was mathematically solved by iteration method for trapezoidal, rectangular, triangular and parabolic channels. The computed values were compared with the graphical method suggested by Chow (1959). The values obtained from graphical method were very close with the values computed by iteration method with variations from -0.20 to 0.40 and -1.38 to 1.00 for sub-critical and super-critical depths, respectively. This justified the correctness and suitability of the iteration method to find out the two alternate depths for a specific energy for different channels.

The specific energy in an open channel section is defined as the energy per unit weight of water at any section measured with respect to the channel bottom. For a channel of small slope and with energy coefficient equal to unity, the specific energy (E) is related to the depth of flow (Y), velocity of flow (V) and acceleration due to gravity (g), and is given by

$$E = Y + \frac{V^2}{2g} \qquad \ldots (1)$$

At critical state of flow, the specific energy is lowest. For a specific energy other than that of critical state, there are two possible depths of flow. These two depths are known as alternate depths (Chow, 1959). One alternate depth greater than the critical depth is known as sub-critical depth, and the other alternate depth less than the critical depth defined as super-critical depth.

Determination of these two alternate depths was first suggested by Chow (1959) through graphical method for rectangular, trapezoidal, triangular and parabolic channel sections. Applying Taylor's theorem of cubic equation, Khanna (1974) determined the two alternate depths of flow for rectangular channel. Nayak *et al.* (1992) found out one alternate depth through iterative method, and the second alternate depth by quadratic equation obtained from the specific energy derivation for rectangular channel section.

In this study, two general equations by iterative method have been presented to compute the alternate depths in trapezoidal, rectangular, triangular and parabolic channels. In the iterative method, to start with, the initial value of alternate depth was considered equal to the critical depth. This method of iteration with initial value of alternate depth equal to critical depth is different from the method of iteration suggested by Nayak *et al.* (1992) for determination of one of the two alternate depths for rectangular channels.

The present iterative method was applied for trapezoidal, rectangular, triangular and parabolic channels, and a comparison has been made for correctness.

MATERIALS AND METHODS

Theoretical Approach

The section factor for critical flow depth (Chow, 1959) is given by

$$Z = A\sqrt{D} \qquad \ldots (2)$$

The hydraulic depth is given as

$$D = \frac{A}{T} \qquad \ldots (3)$$

At critical state of flow (Chow, 1959),

$$Z = \frac{Q}{\sqrt{g}} \qquad \ldots (4)$$

Where,

A = Cross-sectional area of water flow, m^2,

D= Hydraulic depth, m,

Q = Flow rate in the channel, m^3/s,

T = Top width of flow, m,

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g = Acceleration due to gravity, m/s^2, and

Z = Critical flow depth, m.

Inserting the value of D from Eq. (3) in Eqn. (2) and on simplification, one can obtain

$$A = Z^{2/3} . T^{1/3} \qquad \ldots (5)$$

Eqn (5) is a universal equation applicable to all open channels, which shows that the cross-sectional water area at critical state of flow is equal to the product of the two-third power of section factor and one-third power of top width of flow.

Interacting Eqn (4) in Eqn. (5),

$$A = (Q/\sqrt{g})^{2/3} . T^{1/3} \qquad \ldots (6)$$

In the general method for computation of alternate depths, it is necessary to compute the critical depth for the respective channel section with the help of Eqn. (6) using the geometric elements as given in Table 1.

Using the values of A and T from Table 1 in Eqn. (6), the critical depth, Yc, for different channel sections were obtained as follows:

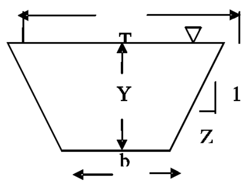
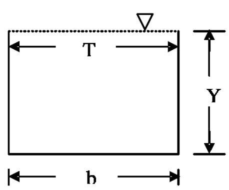
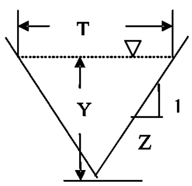
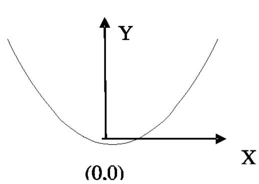
Trapezoidal channel:

$$Yc=\left(\frac{Q^2}{b^2 g}\right)^{1/3} \cdot \left(1+\frac{2Z}{b}Yc\right)^{1/3} \cdot \left(\frac{b}{b+ZYc}\right) \qquad \ldots (7)$$

Rectangular channel:

$$Yc=\left(\frac{Q^2}{b^2 g}\right)^{1/3} \qquad \ldots (8)$$

Table 1. Geometric elements of different channel sections

Section	Area (A)	Top width (T)
Trapezoidal 	(b+ZY)Y	b+2ZY
Rectangular 	bY	b
Triangular (equilateral sides) 	ZY^2	2ZY
Parabolic ($Y= aX^2$)  a = Shape factor	$\frac{4}{3\sqrt{a}}Y^{3/2}$	$\left(\frac{4Y}{a}\right)^{\frac{1}{2}}$

Triangular (equilateral) channel:

$$Yc = \left(\frac{2Q^2}{gZ^2}\right)^{1/5} = 1.148\left(\frac{Q^2}{gZ^2}\right)^{1/5} \quad \ldots (9)$$

Parabolic channel:

$$Yc = \left(\frac{3}{4}\right)^{3/4} \cdot \left(\frac{2Q^2a}{g}\right)^{1/4} = 0.958\left(\frac{aQ^2}{g}\right)^{1/4} \quad \ldots (10)$$

The equation of continuity in any channel section is represented as

$$Q = A.V \quad \ldots (11)$$

Inserting the value of V from eqn. (11), the specific energy equation yields

$$E = Y + \frac{V^2}{2g} = Y + \frac{Q^2}{2gA^2} \quad \ldots (12)$$

or

$$Y = E - \frac{Q^2}{2gA^2} \quad \ldots (13)$$

and

$$E - Y = \frac{Q^2}{2gA^2} \quad \ldots (14)$$

Using the values of water area, A (Table 1) in Eqn. (13) and (14) for different channel sections, the equations for alternate depths, y, obtained are given in Table 2.

The Eqn. (15) to Eqn. (22) presented in Table 2 can be solved for Y by iteration method with initial value of Y equal to Yc. The iteration process is within the capability of scientific calculator. It is to be noted that in general, the solutions obtained from Eqn. (13) i.e., solution of Eqn. (15), (17), (19) and (21) indicate the sub-critical depth, and those obtained from Eqn. (14) i.e., Eqn (16), (18), (20) and (22) yield the super-critical depth for trapezoidal, rectangular, triangular and parabolic channels respectively.

RESULTS AND DISCUSSION

The solutions for determination of the two alternate depths as obtained in Eqn. (15) to Eqn. (22) for different channel sections could be found out by iteration method. The suggested equations were applied to 4 typical cases.

Table 2. Equations for computation of alternate depths for different open channel sections

Channel Section	Equations obtained from Eqn. (13)	Equation obtained from Eqn. (14)
Trapezoidal	$Y = E - \frac{Q^2}{2gb^2Y^2(1+\frac{Z}{b}y)^2}$ -(15) $Y : Yc$	$Y = \frac{Q}{b\sqrt{2g}}\left(\frac{1}{E-Y}\right)^{1/2} * \left(\frac{1}{1+\frac{Z}{b}Y}\right)$ -(16) $Y : Yc$
Rectangular	$Y = E - \frac{Q^2}{2gb^2Y^2}$ -(17) $Y : Yc$	$Y = \frac{Q}{b\sqrt{2g}} * \left(\frac{1}{E-Y}\right)^{1/2}$ -(18) $Y : Yc$
Triangular	$Y = E - \frac{Q^2}{2gz^2Y^4}$ -(19) $Y : Yc$	$Y = \left(\frac{Q}{Z\sqrt{2g}}\right)^{1/2} * \left(\frac{1}{E-Y}\right)^{1/4}$ -(20) $Y : Yc$
Parabolic	$Y = E - \left(\frac{9Q^2a}{32gY^3}\right)$ -(21) $Y : Yc$	$Y = \left(\frac{9Q^2a}{32g(E-Y)}\right)^{1/3}$ -(22) $Y : Yc$

Case 1

A trapezoidal channel with bottom width of 3.0m and side slopes 1:2 (V:H) carries a flow of 5.0 $m^3.s^{-1}$. Compute the alternate depths for specific energy of 1.5m.

Case 2

A rectangular channel with bottom width of 3.0m carries a flow of 5.0 $m^3.s^{-1}$. Compute the alternate depths for specific energy of 1.5m.

Case 3

A triangular channel with side slopes of 1:2 (V:H) carries a flow of 5 $m^3.s^{-1}$. Compute the alternate depths for specific energy of 1.5m.

Case 4

The shape factor of a parabolic channel section is 0.015. It carries a flow of 5 $m^3.s^{-1}$. Compute the alternate depths for a specific energy of 1.5m.

The problems in all the four cases were also solved for the two alternate depths through graphical method as suggested by Chow (1959) for different channel sections and are shown in Figs. 1-4. The computed values of the two alternate depths for specific energy equal to 1.5m for different channels were then compared with the observed values obtained from the graphical solutions given in Table 3. This certified the correctness of the mathematical iteration method for computation of the alternate depths for different channel sections.

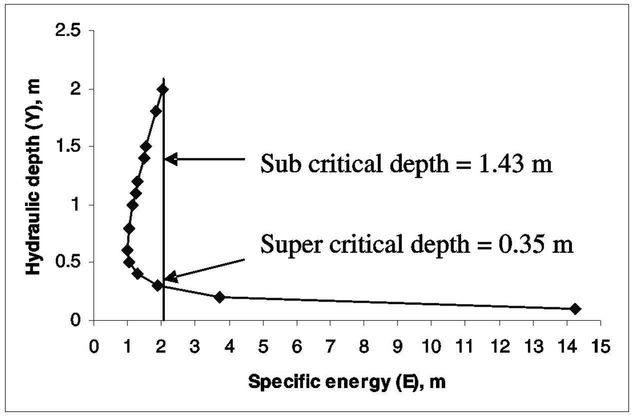


Fig. 2: Relationship between specific energy and hydraulic depth for rectangular section

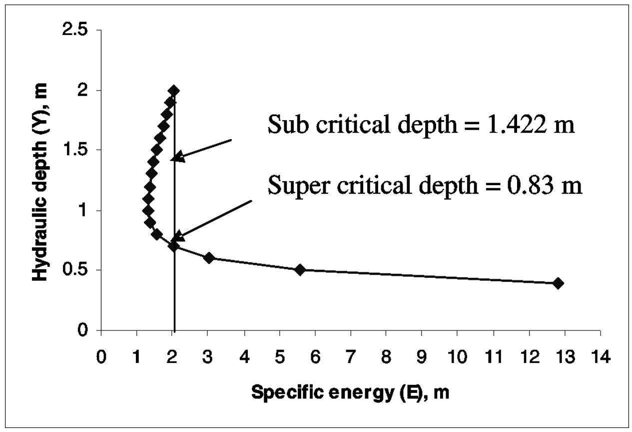


Fig. 3: Relationship between specific energy and hydraulic depth for triangular section

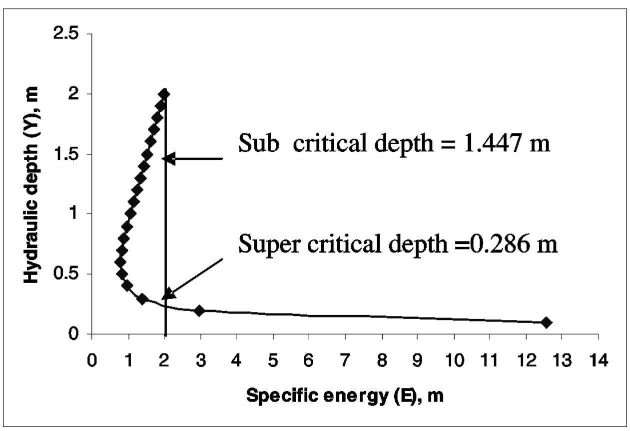


Fig.1: Relationship between specific energy and hydraulic depth for trapezoidal section

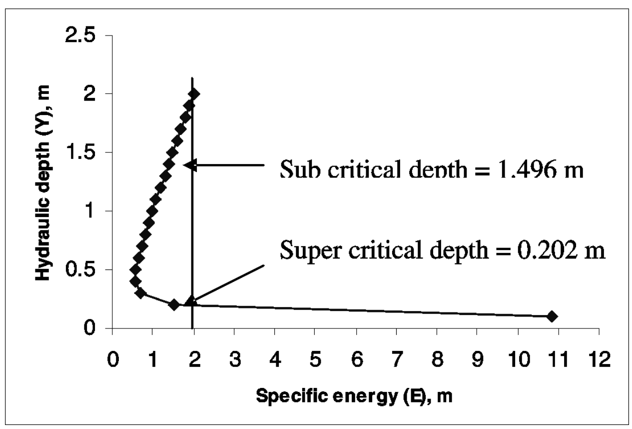


Fig.4: Relationship between specific energy and hydraulic depth for parabolic section

Table 3. Computed and observed alternate depths for different channel sections at specific energy 1.5 m

Channel section	Alternate depths computed by present iteration method (m)		Alternate depths observed from graphical method (m)		Percentage deviation from graphical method value	
	Y1	Y2	Y1	Y2	Y1	Y2
Trapezoidal	1.447	0.286	1.450	0.290	- 0.200	- 1.380
Rectangular	1.430	0.350	1.430	0.350	-	-
Triangular	1.422	0.830	1.420	0.830	0.140	-
Parabolic	1.496	0.202	1.490	0.200	0.400	1.000

CONCLUSIONS

Computation of one alternate depth using cubic equation and second alternate depth through Taylor's theorem for a specific energy (Khanna,1974), determination of one alternate depth through iteration process and second alternate depth by quadratic equation (Nayak *et al.*1922) and the graphical solution for determination of the two alternate depths (Chow, 1959)indicated that the proposed method of mathematical iteration process showed a suitable and simpler method for computation of the two alternate depths for a specific energy for different channel sections.

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