Effect of Setup Cost on Inventory and Service Level

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Abstract

In a batch-manufacturing situation, a reduction in setup cost results in higher marginal reduction in working stock, total variable cost, and safety stock. It also increases the average annual service level. It makes strong economic sense to reduce setup cost from the existing level to the minimum possible level, as each successive reduction brings in higher savings. When setup cost approaches zero, there is a marked reduction in inventory and total variable cost.

Key Words: Setup Cost, Service Level, Inventory, Safety Stock

Section I

1. Introduction

Economic lot size for batch production can help an organisation to cut its inventory to the barest minimum level. The decrease in setup cost reduces inventory and increases service level. Also, irrespective of original setup cost and quantum of its reduction the decrease in setup cost results in marginally increasing returns. When setup cost is reduced to very low levels, i.e. when it approaches zero, reduction in inventory level and total variable cost is very large. Also, slopes of working stock curve, safety stock curve and total cost curve increase dramatically near the zero setup cost. Essential theoretical framework for this is developed in Section II of the paper. Section III demonstrates this with the help of a numerical illustration. Results of this illustration are contained in Tables 1, 2A, 2B and 3.

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Section II

2. Theoretical Framework

Inventory in a batch-manufacturing situation can be divided into two parts. One part depends upon the batch produced and is called working stock. Second part is maintained to take care of fluctuations in demand during the supply lead time and is known as safety stock. The safety stock depends upon the lead time, variability in the lead time and variability of the daily demand during lead time, and desired service level. The lead time in batch manufacturing consists of time spent in initiating the production of the batch, processing time of the batch and time required to make available the output for consumption. If assumption is made that the batch can only be used after it is completely processed (instantaneous replenishment) and that other assumptions of Harris Formula also hold, then average inventory, I, is given by:

$$I = \frac{\text{batch size}}{2} + \text{safety stock}$$

2.1 Effect of Reduction in Setup Cost on Batch Size and Variable Cost

Let us define the following:

Q = economic lot size in units

D = annual expected demand in units

d = daily demand in units

K = setup cost

P = unit cost

h = holding cost per unit per year

C = annual total variable cost

C_s = annual variable cost or holding cost of safety stock component of inventory

C_q = annual variable cost of working stock component of inventory

I = average inventory in units

I_s = safety stock in units

I_q = average working stock in units

= $Q/2$

Then

$I = I_q + I_s$

$C_s = h\, I_s$

$Q = \sqrt{2DK/h}$

$C_q = \sqrt{2DhK}$

$I_q = \frac{1}{2}\sqrt{2DK/h}$ 1

Let

K' = reduced setup cost

Q' = economic lot size when setup cost is K'

= $\sqrt{2DK'/h}$

I_q' = average working stock when setup cost is K'

= $\frac{1}{2}\sqrt{2DK'/h}$

Reduction in average working stock $I_q - I_q'$ is

$= \sqrt{DK/2h} - \sqrt{DK'/2h}$

$= \sqrt{D/2h}\,(\sqrt{K} - \sqrt{K'})$ 2

By differentiating I_q (equation 1) with respect to K we get the slope of the average working stock curve.

$$\frac{dI_q}{dK} = \sqrt{\frac{D}{8h}} \cdot \sqrt{\frac{I}{K}}$$

$$= \text{constant } \sqrt{I/K} \qquad \text{........ 3}$$

similarly

$$\frac{dC_q}{dK} = \sqrt{\frac{Dh}{2}} \cdot \sqrt{\frac{I}{K}} \qquad \text{........ 4}$$

$$= \text{constant } \sqrt{I/K}$$

From equations 3 and 4 it follows that slope of average working stock curve and annual total variable cost curve is proportional to $(I/K)^{1/2}$. This means as K decreases, slope of the average working stock curve increases and as K approaches zero increase in the slope of the curve is rather steep. This shows that the continuous reduction in setup cost results in higher marginal reduction in average working stock. Also, the reduction in average working stock becomes markedly high when setup cost is near zero value. The variable cost component C_q also varies in the similar manner with respect to the changes in setup cost.

2.2 Effect of Reduction in Setup Cost on Safety Stock

The safety stock depends upon the variability of demand during the lead time and service level. Reduction in setup cost reduces the lot size thereby reducing the processing time of the batch. If we assume that the setup cost is proportional to setup time, the reduction in setup cost reduces setup time also. This results in reduction of the variability of demand during the lead time and consequently safety stock requirment for a given service level.

Let us define the following:

L = total lead time in days

t_s = time required to setup the batch in days

t_q = time necessary to process the batch

t_i = time required to make available the output of the batch for use

t_s' = reduced setup time in days

t_q' = time required to process smaller batch when setup cost is K'

L' = reduced lead time due to reduction in setup cost

σ_d = standard deviation of daily demand

R_p = rate of production in units per day

I_s' = safety stock when lead time is L'

Then

$$L = t_s + t_q + t_i \quad \ldots\ldots\ldots 5$$

$$L' = t_s' + t_q' + t_i$$

$$t_q = \frac{\sqrt{2DK/h}}{R_p} \quad \ldots\ldots\ldots 6$$

$$t_q' = \frac{\sqrt{2DK'/h}}{R_p}$$

As

$$t_s \propto K$$

$$t'_s \propto K'$$

$$t_s = \lambda . K \quad (\lambda \text{ is a constant})$$

$$t_s' = \lambda . K'$$

$$= t_s . K'/K$$

$$L = t_s + \sqrt{\frac{2DK}{h}} . \frac{1}{R_p} + t_i$$

$$L' = t_s . \frac{K'}{K} + \sqrt{\frac{2DK'}{h}} . \frac{1}{R_p} + t_i \quad \ldots\ldots\ldots 7$$

Also

$$I_s = Z\sigma_d \sqrt{L}$$

$$I_s' = Z\sigma_d \sqrt{L'}$$

Where Z is number of standard deviations for a certain service level.

Then reduction in safety stock is $I_s - I_s'$

$$= Z\sigma_d\sqrt{L} - Z\sigma_d \sqrt{L'}$$

$$= Z\sigma_d (\sqrt{L} - \sqrt{L'}) \quad \ldots\ldots\ldots 8$$

(Figure 1 is a typical curve showing variation in safety stock with respect to K based on the data of numerical illustration of Section III.)

Also

$$I_s = Z\sigma_d \sqrt{L}$$

$$= Z\sigma_d \sqrt{\lambda K + (2DK/h)^{1/2} .1/R_p + t_i}$$

$$= Z\sigma_d \sqrt{\lambda K + (2D/hR_p^2)^{1/2} . K^{1/2} + t_i} \quad \ldots . 9$$

Differentiating I_s with respect to K, we get slope of the safety stock curve.

$$\frac{dI_s}{dK} = \frac{Z\sigma_d}{2} \left[\frac{1}{\lambda K + (2D/hR_p^2)^{1/2} \cdot K^{1/2} + t_i}\right]^{1/2}$$

$$\times [\lambda + (2D/h R_p^2)^{1/2} . 1/2K^{1/2}]$$

$$= \frac{Z\sigma_d}{2} \left[\frac{1}{\lambda K + (2D/hR_p^2)^{1/2} K^{1/2} + t_i}\right]^{1/2}$$

$$\times [\lambda + (D/2hR_p^2)^{1/2} . 1/K^{1/2}] \quad \ldots\ldots 10$$

2.3 Effect of reduction in setup cost on total variable cost C.

$$C = \sqrt{2hDK} + h . \text{safety stock}$$

$$C = \sqrt{2hDK} + h Z\sigma_d \sqrt{\lambda K+(2D/hR_p^2)^{1/2}.K^{1/2}+t_i}$$

$$\frac{dC}{dK} = \sqrt{\frac{Dh}{2} . \frac{1}{K^2}} + \frac{hZ\sigma_d}{2} \left[\frac{1}{\lambda K + (2D/hR_p^2)^{1/2}.K^{1/2}+t_i}\right]^{1/2}$$

$$\times [\lambda + (D/2hR_p^2)^{1/2} . 1/K^{1/2}] \quad \ldots\ldots 11$$

By substituting the values of D, h, σ_d, Z, t_s, R_p, t_i and K (numerical illustration of section III) in equations 3, 10, and 11, we obtain the values of slopes of working stock curve, safety stock curve and total variable cost curve. Table 1 contains the values of slopes of these curves for various values of K between 0.1 to 1000.

Typical curves representing the slopes of safety stock curve, working stock curve and total variable stock curve are shown in Figure 1. From these curves it is clear that when setup cost decreases slope of these curves go on increasing. Also there is a large increase in the slope when K approaches zero.

Table 1

Setup Cost K $	Slope of Working Stock Curve	Slope of Safety Stock Curve	Slope of Total Variable Cost Curve
1000	25.00	6.69	11.34
950	25.65	6.88	11.64
900	26.35	7.08	11.96
850	27.12	7.31	12.31
800	27.95	7.55	12.69
750	28.87	7.82	13.11
700	29.88	8.12	13.58
650	31.01	8.46	14.10
600	32.27	8.84	14.68
550	33.71	9.28	15.34
500	35.36	9.79	16.10
450	37.27	10.38	16.98
400	39.53	11.09	18.03
350	42.26	11.96	19.30
300	45.64	13.06	20.87
250	50.00	14.51	22.90
200	55.90	16.52	25.67
150	64.55	19.59	29.74
100	79.06	25.00	36.62
90	83.33	26.67	38.67
80	88.39	28.67	41.09
70	94.49	31.14	44.02
60	102.06	34.27	47.68
50	111.80	38.43	52.41
40	125.00	44.27	58.85
30	144.34	53.24	68.38
20	176.78	69.28	84.57
10	250.00	109.54	121.91
1	790.57	515.01	419.23
0.1	2500.00	2275.11	1455.02

2.4 Effect of Reduction in Setup Cost on Service Level

Reduction in setup cost reduces lot size and therefore, for a given annual demand D, increases the number of lots produced in a year. As number of lots increases, safety stock is required as many more number of times in a year. This aparently would seem to reduce average service level over the year. However in practice, reduction in K and increase in number of lots achieves normally higher service level.

Let us define the following:

S_{LA} = average service level for the year

L = total lead time

S_L = service level during lead time corresponding to a certain Z value

N = number of lots per year

W_D = number of working days per year

Then

$$S_{LA} = \frac{(W_D - NL) + NLS_L}{W_D}$$

Section III

Numerical Illustrations

Let

D = 1,000,000 units per year

K = $ 1,000

h = $ 0.20 per unit per year

P = $ 1 per unit (as unit cost is $ 1, inventory in units is same as in dollars)

d = 4,000 units per day (assuming 250 working days per year)

σ_d = 1,000 units

R_p = 10,000 units per day

t_s = 10 days

t_i = 0.1 day

Z = 4 (assuming 99.997 % service level)

S_L = 99.997%

Then

$Q = \sqrt{2DK/h}$

$= \sqrt{\dfrac{2 \times 1{,}000{,}000 \times 1{,}000}{0.2}}$

= 100,000 units

N = number of lots per year

$= \dfrac{1{,}000{,}000}{1{,}00{,}000} = 10$

$I_q = Q/2$

= 50,000 units

t_p = processing time of the batch

$= \dfrac{\text{batch size}}{R_p}$

$= \dfrac{100{,}000}{10{,}000} = 10$ days

Lead time L $= t_s + t_p + t_i$

= 10 + 10 + 0.1

= 20.1 days

Safety Stock $I_s = Z\sigma_d \sqrt{L}$

$= 4 \times 1{,}000 \sqrt{20.1}$

= 17,933 units

Average Inventory I = 50,000 + 17,933

= 67,933 units

Total Variable Cost C $= \sqrt{2DhK} + hI_s$

= 20,000 + 0.2 x 17,933

= $ 23,587

Average Service Level

$$S_{LA} = \frac{(W_D - NL) + NLS_L}{W_D}$$

= 99.997588 %

Let setup cost be decreased to $ 900 from $ 1,000

K' = $ 900

$Q' = \sqrt{\dfrac{2 \times 1{,}000{,}000 \times 900}{0.2}}$ = 94,868 units

N' = number of lots per year when K = $ 900

$= \dfrac{1{,}000{,}000}{94{,}868} = 10.541$

I_q' = 47,434 units

Decrease in working stock $I_q - I_q'$ is

= 50,000 - 47,434

= 2,566 units

t_p' = 94,868/10,000

= 9.4846 days

t_s' = K'/K

$= \dfrac{10 \times 900}{1{,}000}$

= 9 days

L' = 9 + 9.4868 + 0.1

= 18.5868 days

$I_s' = Z\sigma_d \sqrt{L'}$

$= 4 \times 1{,}000 \sqrt{18.5868}$

= 17,245 units

Decrease in safety stock is $I_s - I_s'$

= 17,933 - 17,245 = 688 units

safety stock

$C = \sqrt{2DhK'} + h\, I_s'$

= 18973.7 + 3449

= $ 22,423

Reduction in total variable cost is C -C'

= 23,587 - 22,423

= $ 1164

Average service level when K is $ 900

$$= \frac{(W_D - N'L') + N'L'S_L}{W_D}$$

= 99.997649 %

In this manner incremental reduction in working stock, safety stock and total variable cost as well as change in average service level can be calculated by varying values of K. By keeping increments in K same, incremental variations in

Figure 1

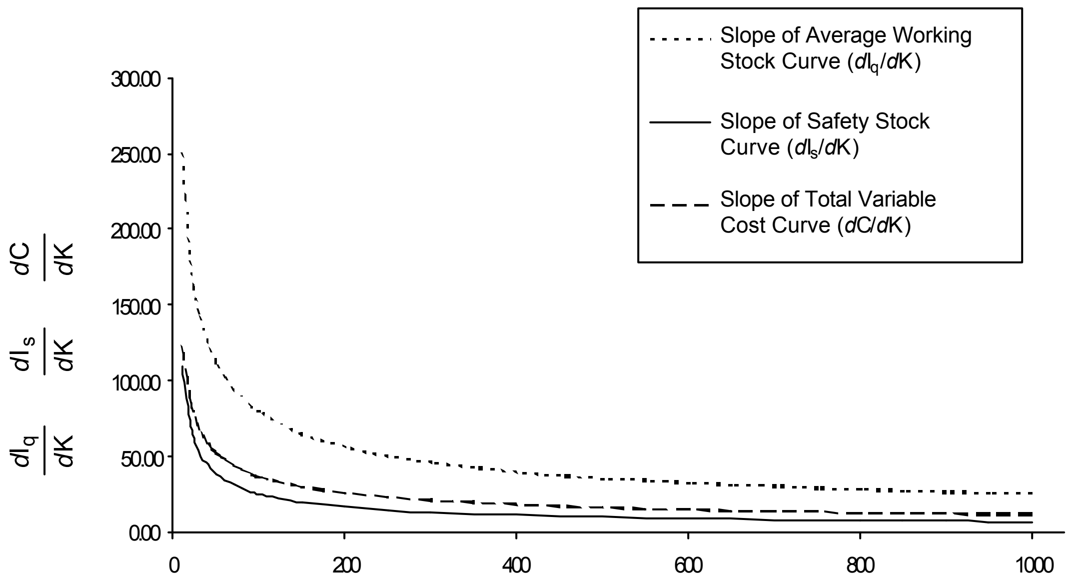

Figure 2

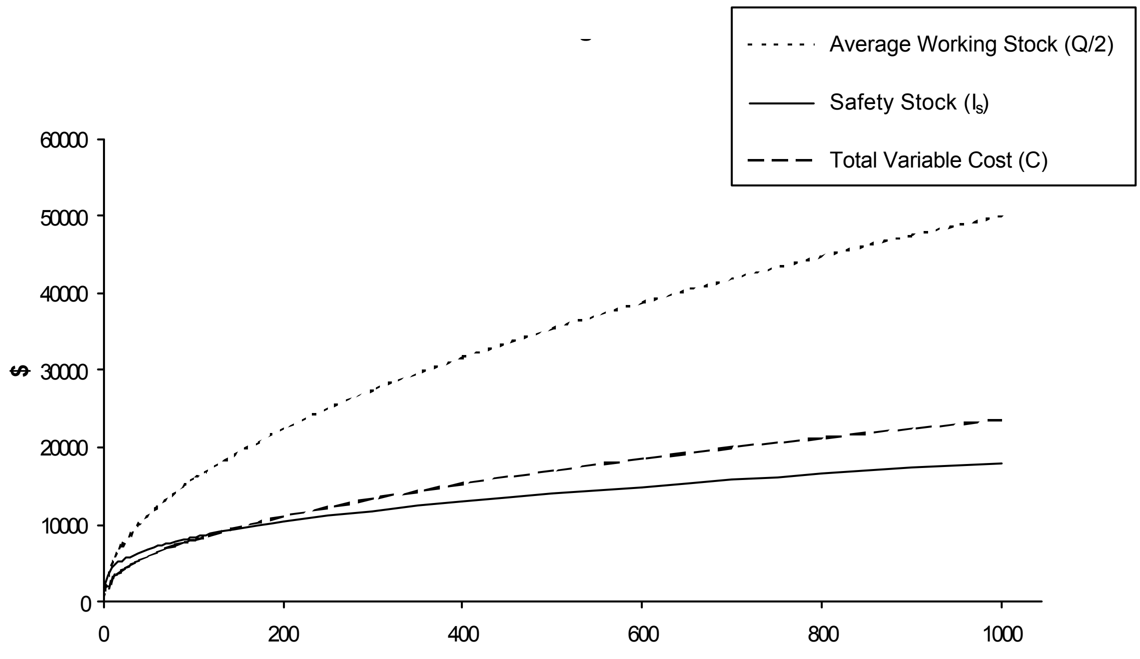

Table 2A

Setup Cost K	Average Working Stock Q/2	Safety Stock I_s	Total Variable Cost C	Incremental Decrease In Working Stock	Incremental Decrease In Safety Stock	Incremental Decrease In Total Variable cost
1000	50000	17933	23587			
900	47434	17245	22423	2566	688	1164
800	44721	16514	21191	2713	731	1231
700	41833	15731	19879	2888	783	1312
600	38730	14884	18469	3103	847	1411
500	35355	13955	16933	3374	929	1536
400	31623	12915	15232	3733	1040	1701
300	27386	11715	13297	4237	1200	1935
200	22361	10254	10995	5025	1460	2302
100	15811	8258	7976	6549	1996	3019
0.1	500	1793	559	15311	6465	7418

Table 2B

Setup Cost K	Average Working Stock Q/2	Safety Stock I_s	Total Variable Cost C	Incremental Decrease In Working Stock	Incremental Decrease In Safety Stock	Incremental Decrease In Total Variable cost
100	15811	8258	7976			
90	15000	8000	7600	811	258	376
80	14142	7724	7202	858	276	398
70	13229	7425	6777	913	299	425
60	12247	7099	6319	981	326	458
50	11180	6736	5819	1067	362	499
40	10000	6325	5265	1180	412	554
30	8660	5841	4632	1340	484	633
20	7071	5237	3876	1589	603	756
10	5000	4382	2876	2071	855	999
0.1	500	1793	559	4500	2588	2318

Note: All figures are in Dollars

Table 3: Effect of Setup Cost on Average Service Level

Setup Cost K ($)	Lead Time L (days)	Average Service Level S_{LA}(%) when t_i = 0.1	Average Service Level S_{LA}(%) when t_i = 0	Probability of Shortage 100-S_{LA}(%) when t_i = 0.1	Probability of Shortage 100-S_{LA}(%) when t_i = 0
1000	20.1000	99.997588	99.997600	0.0024	0.0024
900	18.5868	99.997649	99.997662	0.0024	0.0023
800	17.0443	99.997713	99.997727	0.0023	0.0023
700	15.4666	99.997782	99.997796	0.0022	0.0022
600	13.8460	99.997855	99.997870	0.0021	0.0021
500	12.1711	99.997935	99.997951	0.0021	0.0020
400	10.4246	99.998022	99.998041	0.0020	0.0020
300	8.5772	99.998121	99.998143	0.0019	0.0019
200	6.5721	99.998237	99.998263	0.0018	0.0017
100	4.2623	99.998383	99.998421	0.0016	0.0016
90	4.0000	99.998400	99.998440	0.0016	0.0016
80	3.7284	99.998418	99.998461	0.0016	0.0015
70	3.4458	99.998437	99.998483	0.0016	0.0015
60	3.1495	99.998457	99.998506	0.0015	0.0015
50	2.8361	99.998478	99.998532	0.0015	0.0015
40	2.5000	99.998500	99.998560	0.0015	0.0014
30	2.1321	99.998523	99.998592	0.0015	0.0014
20	1.7142	99.998545	99.998630	0.0015	0.0014
10	1.2000	99.998560	99.998680	0.0014	0.0013
5	0.8571	99.998545	99.998715	0.0015	0.0013
1	0.4262	99.998383	99.998762	0.0016	0.0012
0.1	0.2010	99.997588	99.998788	0.0024	0.0012

I_q, I_s and C can be analysed. Tables 2A & 2B contain values of I_q, I_s, C and incremental variation in I_q, I_s and C for different values of K. Figure 2 graphically shows variation in I_q, I_s and C with respect to K.Table 3 contains average service level and probability of shortage for different values of K.

4.1 Limitations

In the present work it has been assumed that setup cost and setup time are directly related. No research studies were found to substantiate or contradict this assumption. As setup cost mainly depends upon the setup time of the equipment and related manpower, this assumption is logical and is therefore made.

4.2 Conclusion

Reduction in setup cost is essential in reducing inventories, and total variable cost associated with inventories including that of the safety stocks.

Each successive reduction in setup cost provides higher and higher returns. In the vicinity of zero setup cost, reduction in inventory and total variable costs are very large.

In the typical case of Section III, when setup cost decreases from \$ 1,000 to \$ 10, average service level goes on increasing from 99.997588% to 99.998560% (Table 3). Further reduction in K decreases service level from 99.998560% to 99.997588% (at K = \$ 0.1). However, when t_i is zero, then S_{LA} is 99.997600% when K is \$ 1000, and as K decreases from \$ 1000 to \$ 0.1, S_{LA} goes on increasing. When K is \$ 0.1 average service level is 99.998788%. Also probability of shortage becomes almost half from 0.0024% to 0.0012%. From this, conclusion can be drawn that reduction in setup cost has favourable effect on average service level. However when t_i is 0.1 day, average service level starts decreasing after reaching a maximum value of 99.998560% at K = \$ 10. This is because when K becomes very small, value of t_i becomes a substantial component of lead time. At this stage in order to improve average service level reduction in setup time and batch processing time is not enough. The time required to make available the batch for consumption should also be reduced.

4.3 End Notes

In the paper standard expressions for economic lot size, working stock, average inventory, lead time, safety stock and service level are used and these are available in standard texts.