

An analytical solution for contaminant transport against the flow with periodic boundary condition in one-dimensional porous media

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ABSTRACT

The transport of pollutants in porous media, is of hydrodynamic dispersion phenomena, has been the major research subject for more than four decades. The present paper is an attempt to describe analytically the solute transport in one-dimensional homogeneous porous medium of finite domain. The effects of time-periodic pulse-type inlet contaminant concentration and groundwater flow velocity against the contaminant transport are investigated analytically. The temporally dependent dispersion proportional to seepage velocity and retardation factor is also taken into account. The governing transport equation is solved analytically by employing Laplace transformation technique. The effects of various physical parameters on solute transport and their importance are discussed and are also illustrated graphically.

Keywords: Dispersion, pollutant, retardation factor, seepage flow.

1. Introduction

From several decades, the factory activities and uncontrolled use of pesticides in agriculture cause serious damages on the environment in the area and affected groundwater table. The characteristics of pollutants level and transport in groundwater are associated with variations in one or two parameters at one scale and several parameters at another scale. Pollutant behavior in the soil system is subject to many processes. The water soil system is dynamic and many chemical, biological and physical reaction occurs which may affect pollutant behavior. Solute transport inside the aquifer systems is due to hydrodynamic dispersion. Mathematical model is one of the powerful tools to predict the effect of pollution on the long term water resource quality. Most existing analytical solutions for advection–diffusion transport problems including sorption, retardation and decay terms are for semi-infinite or infinite regions. The dispersion coefficient during solute transport in the aquifer was studied by Ogata and Banks (1961), Hoopes and Harleman (1967), Gelhar and Collins (1971). They assumed that the aquifer consists of a geologically homogeneous porous medium. Kumar (1983) presented time-dependent input concentration against exponentially decreasing unsteady velocity. Frind (1982) presented a finite element solution using a velocity-dependent dispersion coefficient and discussed the errors caused by use of a constant dispersion coefficient. Volker and Rushton (1982) compared a variety of aquifer parameters and the influence of the flow conditions on the configuration and location of the interface.

Many studies assume a steady state flow field. However, natural flow systems are rarely in a steady state. Temporal fluctuations in the recharge or in the boundary conditions lead to variations in the velocity field that contribute to dispersive mixing. Field studies of the groundwater discharge process in unconfined coastal aquifers show that the tide can

significantly influence the temporal and spatial patterns of groundwater discharge as well as the salt concentration in the near-shore groundwater (Robinson et al. 1998; Groundwater fluctuations as induced by oceans oscillations affect water and mass exchange between aquifer and ocean (Li et al. 1999,1999a). The period of the fluctuations of the downstream water level affects the oscillations of the hydraulic heads and therefore the velocities within the aquifer. The effect of flow oscillations on solute transport in tubes has been investigated by several researchers. Since contaminants in porous medium migrate with ground water flow, any factors that may affect groundwater flow are also likely to influence the migration of contaminants in porous media. Time periodic convection leading to the dispersion of Brownian particle in solution may be observed in the transport of contaminant in tidal estuaries (Bowden; 1965), the spread tracer in blood vessels (Caro; 1966), the mixing of salt water with fresh water in coastal aquifer (Bear; 1972). Watson (1983) determined that solute dispersion effects along a pipe due to steady and oscillatory flow are additive. Oscillatory flow has also been found to influence dispersion in tubes. Fang et al. (1972) simulate the tidal fluctuation of the groundwater table, by using a two-dimensional finite element model. Custodio (1988) discussed that in many real situations, such as sluggish freshwater flow, stresses caused by tidal oscillations and recharge events, and enhanced dispersivity by macroscopic heterogeneities. Townley (1993, 1995) presented a periodic finite element model called AQUIFEM-P that computes periodic fluctuations in heads and velocities in a 2-D region of aquifer. Logan and Zlotnik (1995) obtained solutions of the convection–diffusion equation with decay for periodic boundary conditions on a semi–infinite domain. The boundary conditions take the form of a periodic concentration or a periodic flux, and a transformation is obtained that relates the solutions of the two, pure boundary value problems. Krabbenhoft and Webster (1995) study the influence of periodic flow reversals on temporal variation in the water chemistry of a shallow seepage lake in Michigan. Ataie-Ashtiani (1998) validated the model for simulation of groundwater flow in response to the periodic boundary condition in unconfined aquifers with a mild sloping face. Andric`evic´ and Cvetkovic´ (1998) reported the solute flux as a function of travel time and transverse displacement in the relative dispersion frame work by removing the plume meandering.

Many analytical solutions have been derived to describe the periodic groundwater flow (Li et al.; 2001, Jeng et al.; 2002). Common problems include salt-water intrusion due to over pumping of groundwater and brine discharges from desalination plants, as well as coastal water pollution by plume leachate from contaminated coastal aquifers (Purnalna et al.; 2003, Masciopinto; 2006,). Song et al. (2007) presented a new perturbation solution of the non-linear Boussinesq equation for one-dimensional tidal groundwater flow in a coastal aquifer. Jaiswal et al. (2009) and Kumar et al. (2010) obtained analytical solutions for temporally and spatially dependent solute dispersion in a one dimensional semi-infinite porous medium. Yadav et al. (2010) obtained analytical solutions of one-dimensional advection–dispersion equation in semi-infinite longitudinal porous domain.

In most hydrological situations contaminant and groundwater flow may vary with time. Contaminant input may varying frequently, weekly, monthly or even on annual basis. Similarly, groundwater flow might be considered likely vary periodically as a result of downfall patterns. The objective of this paper is to address the influence of periodic seepage velocity on the porous medium and consequently on the transport of solute under periodic condition. Also, developed an analytical solution which is represent contaminant concentrations in groundwater systems under periodic seepage flow velocity. In present study, non-reactive pollutants are considered in finite Porous domain. The direction of periodic seepage flow is from $x = 0$ to $x = L$. A source concentration (periodic pulse type) is

enforced at the boundary $x = L$, i.e., against the flow while mixed type boundary condition is assumed at $x = L_1$. Initially the domain is not solute free. Retardation factor is also considered. Laplace Transformation technique (LTT) is used to get the analytical result of the proposed problem.

2. Mathematical formulation and analytical solution

The transport of solutes in saturated, homogeneous porous media, accounting for one-dimensional hydrodynamic dispersion is governed by the following partial differential equation,

$$R \frac{\partial c}{\partial t} = \frac{\partial}{\partial x} \left(D(x, t) \frac{\partial c}{\partial x} - u(x, t) c \right)$$

(1) where $R = \left(1 + \frac{1-n_p}{n_p} K_1\right)$ is a retardation factor accounting for equilibrium linear sorption processes and describing solute sorption, n_p is porosity, K_1 is empirical constant. The use of equilibrium isotherms assumes that equilibrium exists at all times between the porous medium and the solute in solution. This assumption is generally valid when the adsorption process is fast in relation to the ground-water velocity. c is the solute concentration in the liquid phase at position x and time t . The dispersion coefficient, D presumably includes the effects of both molecular diffusion and mixing in the axial direction, however molecular diffusion is negligible due to very low seepage velocity. In Eq. (1), D and u may be constants or function of time. If both the parameters are independent to independent variables x and t , then these are called constant dispersion and uniform flow velocity respectively. The term on the left side of the equal sign indicate the retardation factor and change of concentration in time, the first two terms on the right side describe hydrodynamic dispersion and groundwater velocity.

Let the porous domain is of finite length. Initially the domain is not solute free, it is exponentially increasing function of space variable. A pulse type input contaminants are being introduced till time $t \leq t_0$, where t_0 the time of illumination of source concentration, beyond that source becomes zero. Contaminants will move against the direction of flow and spread out with time due to higher concentration. On other hand, mixed type boundary condition is assumed at $x = L_1$. This type of situation generally exist saline water intrusion into coastal aquifer. Under these assumptions mathematically, initial and boundary conditions of proposed problem can be written as,

$$c(x, t) = C_i \exp(\alpha x), \quad L_1 \leq x \leq L, \quad t = 0$$

(2)

$$c(x, t) = \begin{cases} C_0 \{1 + \cos(mt)\}, & 0 < t \leq t_0 \\ 0, & t > t_0 \end{cases}, \quad x = L$$

(3a)

$$\frac{\partial c}{\partial x} = \frac{u_0}{2D_0} c, \quad x = L_1, \quad t \geq 0$$

(3b)

where C_i is the resident concentration and α is a constant which is less than one and its dimension is inverse of space variable x . Eq. (2) shows that the initial condition is an increasing exponentially function of x and m unsteady coefficient of dimension of inverse of time t . An input concentration of periodic nature is assumed at the one point of the domain i.e., $x = L$. The field observations indicate the source concentration may not be negative, therefore $C_0 \{1 + \cos(mt)\}$ is taken. The coefficient of dispersion is considered directly proportional to seepage velocity (Yim and Mohsen, 1992), i.e.,

$$D(x, t) \propto |u(x, t)|.$$

Let us write $u(x, t) = u_0 |\sin(mt)|$, so that $D(x, t) = D_0 |\sin(mt)|$, where D_0 and u_0 are dimension of D_0 is $L^2 T^{-1}$ hence it is initial diffusion coefficient. But initial diffusion coefficient D_0 is zero at $t = 0$. But $D(x, t) = D_0$ at $t = \pi/2$ i.e., D_0 represents the peak value of $D_x(t)$. Similarly u_0 may be interpreted, and the dimension of u_0 is LT^{-1} , respectively. m is a unsteady parameter whose dimension is inverse to that the time variable t . At $t = 0$, D_0 and u_0 are equal to zero, so Eq. (1) is valid in $t > 0$ domain. Eq. (1) may be written in absolute form as,

$$R \frac{\partial c}{\partial t} = \frac{\partial}{\partial x} \left(D_0 |\sin(mt)| \frac{\partial c}{\partial x} - u_0 |\sin(mt)| c \right) \quad (4)$$

Let us introduce a new time variable using the following transformation (Crank, 1975),

$$T = \int_0^t |\sin(mt)| dt$$

or $T = \int_0^t \sin(mt) dt$ as the $|\sin(mt)| = \sin(mt)$ in $t > 0$ domain.

or $mT = 1 - \cos(mt)$

$$(5)$$

Now differential equation (4) reduces into constant coefficients as

$$R \frac{\partial c}{\partial T} = \frac{\partial}{\partial x} \left(D_0 \frac{\partial c}{\partial x} - u_0 c \right) \quad (6)$$

The conditions in terms of new time variable may be written as

$$c(x, T) = C_i \exp(\alpha x), \quad L_1 \leq x \leq L, \quad T = 0 \quad (7)$$

$$c(x, T) = \begin{cases} C_0(2 - mT), & 0 < T \leq T_0 \\ 0, & T > T_0 \end{cases}, \quad x = L \quad (8a)$$

$$\frac{\partial c}{\partial x} = \frac{u_0}{2D_0} c, \quad x = L_1, \quad T \geq 0 \quad (8b)$$

Now introducing a new dependent variable by following transformation

$$c(x, T) = K(x, T) \exp \left[\frac{u_0}{2D_0} x - \frac{u_0^2}{4RD_0} T \right] \quad (9)$$

The set of Eqs. (6), (7) and (8a,b) reduced into

$$R \frac{\partial K}{\partial T} = D_0 \frac{\partial^2 K}{\partial x^2} \quad (10)$$

$$K(x, T) = C_i \exp(\alpha x - \beta x), \quad L_1 \leq x \leq L, \quad T = 0 \quad (11)$$

$$K(x, T) = \begin{cases} C_0(2 - mT) \exp(-\beta x + \gamma^2 T), & 0 < T \leq T_0 \\ 0, & T > T_0 \end{cases}, \quad x = L \quad (12a)$$

where $\beta = \frac{u_0}{2D_0}$, $\gamma^2 = \frac{u_0^2}{4RD_0}$

$$\frac{\partial K}{\partial x} = 0, \quad x = L_1, \quad T > 0 \quad (12b)$$

Applying Laplace transformation on equations (10) – (12), we have

$$(13) \quad Rp\bar{K} - C_i R \exp(\alpha x - \beta x) = D_0 \frac{d^2 \bar{K}}{dx^2}$$

$$\bar{K}(x, p) = C_0 \exp(-\beta L) \left[\frac{1}{(p-\gamma^2)} \{2 - (2 - mT_0) \exp(-(p-\gamma^2)T_0)\} \right. \\ \left. - \frac{m}{(p-\gamma^2)^2} \{1 - \exp(-(p-\gamma^2)T_0)\} \right], x = L \quad (14a)$$

$$(14b) \quad \frac{d\bar{K}}{dx} = 0, \quad x = L_1$$

Thus the general solution of equation (13) may be written as

$$(15) \quad \bar{K}(x, p) = C_1 \cosh(Mx) + C_2 \sinh(Mx) + \frac{C_i \exp(\alpha - \beta)x}{\left\{p - \frac{D_0}{R}(\alpha - \beta)^2\right\}}; \quad M = \sqrt{\frac{Rp}{D_0}}$$

Using conditions (14a,b) on the above solution, we get

$$C_1 = C_0 \exp(-\beta L) \left[\frac{1}{(p-\gamma^2)} \{2 - (2 - mT_0) \exp(-(p-\gamma^2)T_0)\} \right. \\ \left. - \frac{m}{(p-\gamma^2)^2} \{1 - \exp(-(p-\gamma^2)T_0)\} \right] \frac{\cosh ML_1}{\cosh(LM - L_1 M)} \\ - \frac{C_i}{\left\{p - \frac{D_0}{R}(\alpha - \beta)^2\right\}} \left\{ \frac{\exp p\{(\alpha - \beta)L\} \cosh ML_1}{\cosh(LM - L_1 M)} - \frac{(\alpha - \beta) \exp p\{(\alpha - \beta)L_1\} \sinh ML}{M \cosh(LM - L_1 M)} \right\} \quad (16)$$

and

$$C_2 = -C_0 \exp(-\beta L) \left[\frac{1}{(p-\gamma^2)} \{2 - (2 - mT_0) \exp(-(p-\gamma^2)T_0)\} \right. \\ \left. - \frac{m}{(p-\gamma^2)^2} \{1 - \exp(-(p-\gamma^2)T_0)\} \right] \frac{\sinh ML_1}{\cosh(LM - L_1 M)} \\ - \frac{C_i}{\left\{p - \frac{D_0}{R}(\alpha - \beta)^2\right\}} \left\{ \frac{(\alpha - \beta) \exp p\{(\alpha - \beta)L\} \cosh ML}{M \cosh(LM - L_1 M)} - \frac{\exp p\{(\alpha - \beta)L\} \sinh ML_1}{\cosh(LM - L_1 M)} \right\} \quad (17)$$

Thus the solution in the Laplacian domain may be written as

$$\bar{K}(x, p) = \left[C_0 \exp(-\beta L) \left\{ \frac{1}{(p-\gamma^2)} \{2 - (2 - mT_0) \exp(-(p-\gamma^2)T_0)\} \right. \right. \\ \left. \left. - \frac{m}{(p-\gamma^2)^2} \{1 - \exp(-(p-\gamma^2)T_0)\} \right\} \frac{\cosh ML_1}{\cosh(LM - L_1 M)} \right. \\ \left. - \frac{C_i}{\left\{p - \frac{D_0}{R}(\alpha - \beta)^2\right\}} \left\{ \frac{\exp p\{(\alpha - \beta)L\} \cosh ML_1}{\cosh(LM - L_1 M)} - \frac{(\alpha - \beta) \exp p\{(\alpha - \beta)L_1\} \sinh ML}{M \cosh(LM - L_1 M)} \right\} \right] \cosh(Mx) \\ - \left[C_0 \exp(-\beta L) \left\{ \frac{1}{(p-\gamma^2)} \{2 - (2 - mT_0) \exp(-(p-\gamma^2)T_0)\} \right. \right. \\ \left. \left. - \frac{m}{(p-\gamma^2)^2} \{1 - \exp(-(p-\gamma^2)T_0)\} \right\} \frac{\sinh ML_1}{\cosh(LM - L_1 M)} \right. \\ \left. - \frac{C_i}{\left\{p - \frac{D_0}{R}(\alpha - \beta)^2\right\}} \right]$$

$$\left\{ \frac{(\alpha-\beta) \exp\{(\alpha-\beta)L_1\} \cosh ML - \exp\{(\alpha-\beta)L\} \sinh ML_1}{M \cosh(LM-L_1M)} - \frac{\exp\{(\alpha-\beta)L\} \sinh ML_1}{\cosh(LM-L_1M)} \right\} \sinh(Mx) + \frac{C_i \exp(\alpha-\beta)x}{\left\{p - \frac{D_0}{R}(\alpha-\beta)^2\right\}}$$

or $\bar{K}(x,p) = \left[C_0 \exp(-\beta L) \left\{ \frac{1}{(p-\gamma^2)} \{2 - (2 - mT_0) \exp(-(p-\gamma^2)T_0)\} \right. \right.$

$$- \frac{m}{(p-\gamma^2)^2} \{1 - \exp(-(p-\gamma^2)T_0)\} \left. \left\{ \frac{\cosh(Mx-ML_1)}{\cosh(LM-L_1M)} \right. \right.$$

$$\left. \left. - \frac{C_i}{\left\{p - \frac{D_0}{R}(\alpha-\beta)^2\right\}} \left\{ \frac{\exp\{(\alpha-\beta)L\} \cosh(Mx-ML_1)}{\cosh(LM-L_1M)} - \frac{(\alpha-\beta) \exp\{(\alpha-\beta)L_1\} \sinh(Mx-ML)}{M \cosh(LM-L_1M)} \right\} \right. \right.$$

$$\left. \left. + \frac{C_i \exp(\alpha-\beta)x}{\left\{p - \frac{D_0}{R}(\alpha-\beta)^2\right\}} \right] \quad (18)$$

Taking inverse Laplace transform of equation (18) which are discussed in Appendix, the solution of advection-dispersion solute transport for periodic input condition in terms of $c(x,T)$ as,

$$c(x,T) = C_i F_1(x,T) + 2C_0 F_2(x,T) + C_0 F_3(x,T); 0 < T \leq T_0 \quad (19a)$$

$$c(x,T) = C_i F_1(x,T) + C_0 [2F_2(x,T) - (2 - mT_0)F_2(x,T - T_0)] + C_0 [F_3(x,T) - F_3(x,T - T_0)]; T > T_0 \quad (19b)$$

where $F_1(x,T) = \exp\{\alpha x + (b - \gamma^2)T\}$

$$- \exp(\alpha L - \beta L + \beta x - \gamma^2 T) \left\{ 2e^{bT} \frac{\cosh(x-L_1)\sqrt{bR/D_0}}{\cosh(L-L_1)\sqrt{bR/D_0}} - 2\pi D_0 \sum_{n=0}^{\infty} \frac{(-1)^n (n+1/2) \cos\{(n+1/2)\pi \frac{(x-L_1)}{(L-L_1)}\}}{\{(n+1/2)^2 \pi^2 D_0 + bR(L-L_1)^2\}} \exp\left\{-\frac{(n+1/2)^2 \pi^2 D_0}{R(L-L_1)^2} T\right\} \right\}$$

$$- (\alpha - \beta) \exp(\alpha L_1 - \beta L_1 + \beta x - \gamma^2 T) \left\{ \frac{e^{bT}}{\sqrt{bR/D_0}} \frac{\sinh(x-L)\sqrt{bR/D_0}}{\cosh(L-L_1)\sqrt{bR/D_0}} - 2(L-L_1)D_0 \sum_{n=0}^{\infty} \frac{(-1)^n \sin\{(n+1/2)\pi \frac{(x-L)}{(L-L_1)}\}}{\{(n+1/2)^2 \pi^2 D_0 + bR(L-L_1)^2\}} \exp\left\{-\frac{(n+1/2)^2 \pi^2 D_0}{R(L-L_1)^2} T\right\} \right\},$$

$$F_2(x,T) = \exp\{(x-L)\beta\} \frac{\cosh(x-L_1)\beta}{\cosh(L-L_1)\beta} - 2\pi D_0 \exp\{(x-L)\beta - \gamma^2 T\} \times$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{(n+1/2) \cos\{(n+1/2)\pi \frac{x-L_1}{L-L_1}\}}{\{(n+1/2)^2 \pi^2 D_0 + \gamma^2 R(L-L_1)^2\}} \exp\left\{-\frac{(n+1/2)^2 \pi^2 D_0}{R(L-L_1)^2} T\right\},$$

$$F_3(x,T) = \exp\{(x-L)\beta\} \left[\left\{ (L-L_1) \frac{mR}{u_0} \tanh(L-L_1)\beta - mT \right\} \frac{\cosh(x-L_1)\beta}{\cosh(L-L_1)\beta} - \frac{mR(x-L_1) \sinh(x-L_1)\beta}{u_0 \cosh(L-L_1)\beta} \right]$$

$$-2\pi D_0 \exp\{(x-L)\beta - \gamma^2 T\} \sum_0^\infty (-1)^n \left\{ \frac{mR(L-L_1)^2 (n+1/2) \cos\left\{(n+1/2)\pi \frac{x-L_1}{L-L_1}\right\}}{\left(\{(n+1/2)^2 \pi^2 D_0 + \gamma^2 R(L-L_1)^2\}^2\right)^{1/2}} \right\} \exp\left\{-\frac{(n+1/2)^2 \pi^2 D_0}{R(L-L_1)^2} T\right\},$$

$$b = \frac{D_0(\alpha-\beta)^2}{R}, \quad \beta = \frac{u_0}{2D_0}, \quad \gamma^2 = \frac{u_0^2}{4RD_0} \quad \text{and} \quad T = \frac{1}{m}\{1 - \cos(mt)\}.$$

3. Results and Discussion

The effect of the model parameters, periodic seepage velocity, dispersion coefficients, retardation coefficients on the transport of dissolved substance are graphically presented here by different graphs and table. Some of the parameters used in the examples (numerical values) can be found in various models available in the literature. (Gelhar and Collins;1971, Logan and Zlotnik;1995, Jaiswal et al.;2009, Kumar et al.; 2010, etc.). An important assumption in the study is that the flow velocity and boundary conditions are periodic. The graphical representation of solutions (19a) and (19b) are presented by different figures. All concentrations are conveniently express as dimensionless quantity $\frac{C}{C_0}$ with C_0 being equal to unity. The graphs of relative concentration as ordinate and distance as abscissa have been plotted, to know the variation of concentration distribution in space domain. All figures are computed for various parameters used in the present model with a short domain length of $0 \leq x(\text{m}) \leq 10$. To demonstrate the concentration profiles of the present problem, the values of the parameters used are: $D_0 = 1.66$ (m²/day), $u_0 = 1.32$ (m/day), $C_0 = 1.0$, $C_i = 0.1$ and $\alpha = 0.032$ and unsteady coefficient m (day⁻¹) = 0.1. The groundwater velocity may vary from 2 (m/day) to 2 (m/year) depending upon geometrical conditions of porous medium (Todd, 1980).

Figure (1) is drawn in the time domain $t \leq t_0$ (in presence of source pollutant) at t (day) = 48, 60, 72, and Fig. (2) is drawn in the domain $t > t_0$ (in absence of source pollutant) at t (day) = 81, 93, 105. The elimination time (t_0) of source pollutant is $t_0 = 80$ (day). The time interval is considered to be 12 days. Other parameters, R and m are taken 1.45 and 0.1 (day⁻¹), respectively. In figure (1), it may be observed that in the presence of a source pollutant, concentration values rapidly decrease with position near the inlet boundary. At the inlet boundary concentration changes periodically with time. Periodic behavior also observed at the other end of the domain. It may be also observed that in the absence of a source pollutant (Figure 2), concentration values rapidly increase up to the position $x = 8$ (approximately), but periodic nature also maintain throughout the domain. In both cases ($t \leq t_0$ and $t > t_0$) concentrations values are highly depends on time.

Figures (3) and (4) illustrate the concentration profiles by surface graph for solution (19a, b). Common parameters are used as $m = 0.1$ (day⁻¹) and $R = 1.45$. Figures (5) and (6), demonstrates the concentration profiles for solution (19a, b) in time domain $t \leq t_0$ (in the presence of source pollution) and $t > t_0$ (in the absence of source pollution) for different unsteady coefficients m (day⁻¹) = 0.08, 0.1, 0.12 and time t (day) = 60 and 81, respectively.

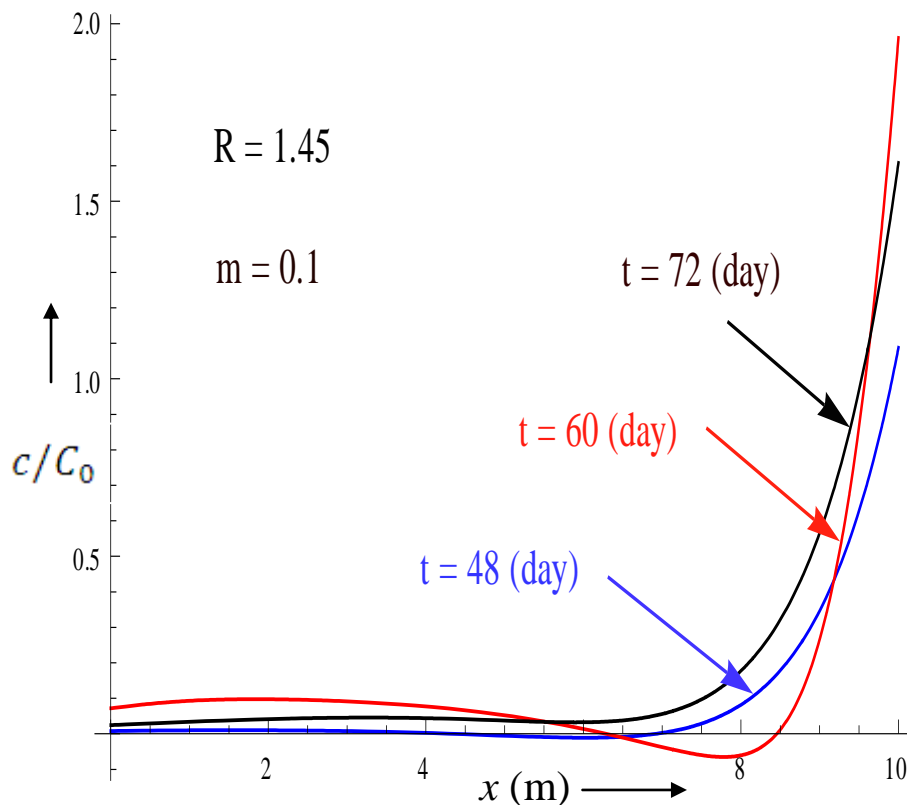


Figure 1: Distribution of the dimensionless concentration at various time and fix retardation factor in presence of source concentration.

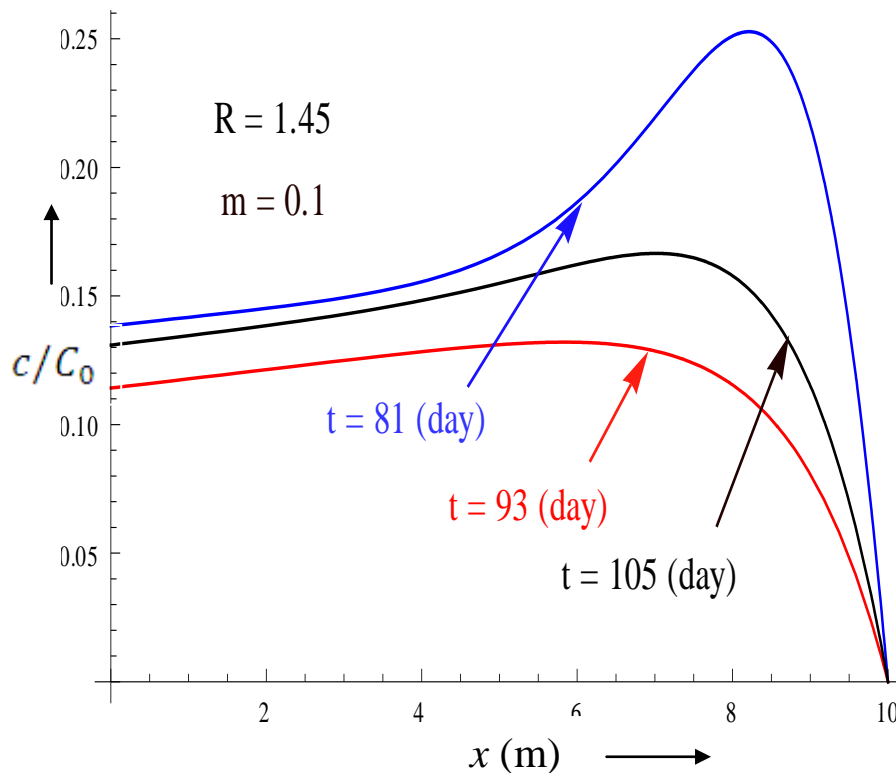


Figure 2: Distribution of the dimensionless concentration at various time and fix retardation factor in absence of source concentration.

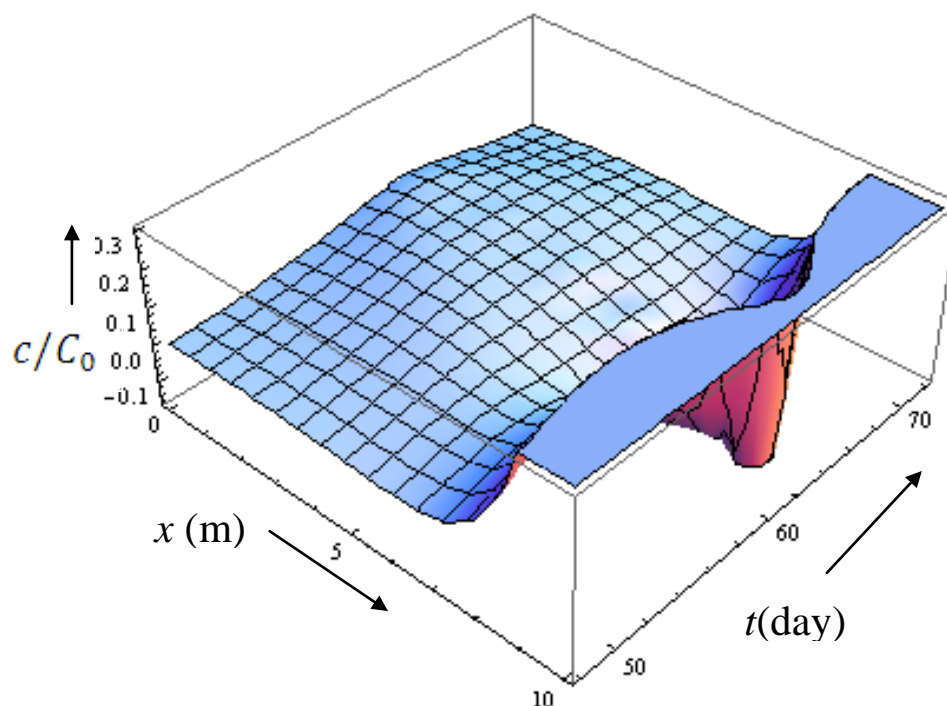


Figure 3: Surface plot of the dimensionless concentration in presence of source concentration.

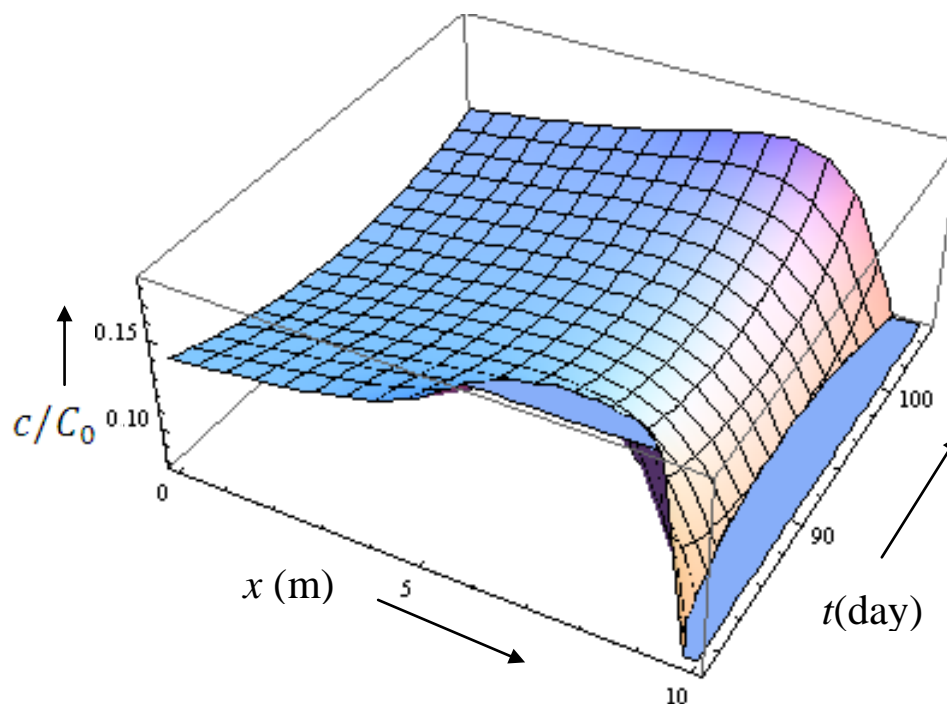


Figure 4: Surface plot of the dimensionless concentration in absence of source concentration.

Common retardation factor $R = 1.45$ is taken in both situations. The concentration values are periodically changes with increasing unsteady coefficient in both time domains. It may be also observed that in time domain $t \leq t_0$ concentration values decreases rapidly near the source boundary while in $t > t_0$ it increases. Concentration values in time domain $t \leq t_0$ is slightly higher than $t > t_0$ on other boundary. Concentration behaviors are also depend on unsteady coefficient m .

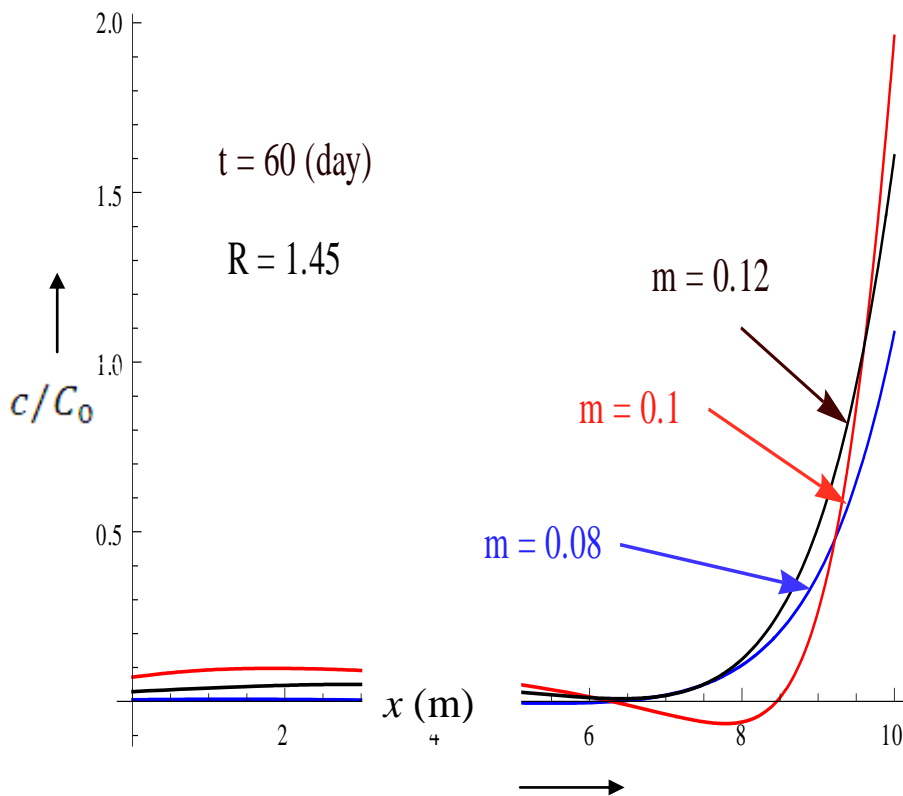


Figure 5: Distribution of the dimensionless concentration at various unsteady coefficient at fix time and retardation factor in presence of source concentration.

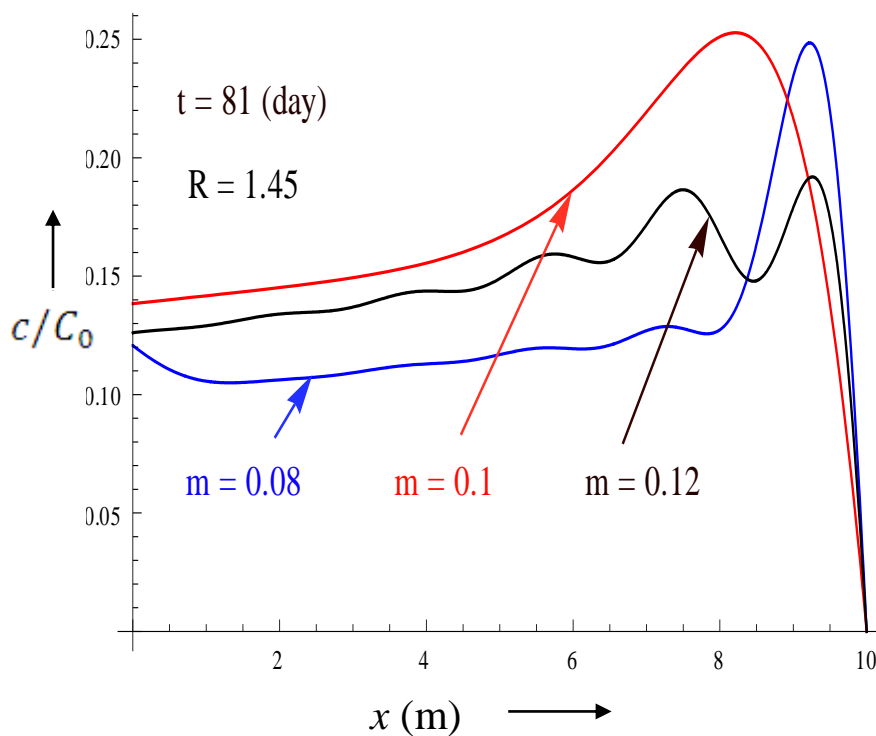


Figure 6: Distribution of the dimensionless concentration at various unsteady coefficient at fix time and retardation factor in absence of source concentration.

Table-1: Values of new time T with respect to old time t (day) where $T = \frac{1}{m}\{1 - \cos(mt)\}$.

t (day)	T	t (day)	T	t (day)	T
00	00.000	62.82	00.000	125.64	000
10.47	04.998	73.29	04.988	136.11	977
20.94	14.996	83.76	14.986	146.58	976
31.41	20.000	94.23	20.000	157.05	000
41.88	15.006	104.70	15.017	167.52	027
52.35	05.008	115.17	05.018	177.99	029

Table-1, illustrates the periodic relation between old time t and new time T . It is observed that, after every time interval $t(\text{day}) = 62.82$, the concentration profiles periodically attained the same value in the domain. Once the source of the pollution is removed, the input concentration becomes zero and, at a particular position concentration level, starts decreasing with time and appears to settle at a minimum level. Once the source is removed, the rehabilitation process is almost completed in a finite time period.

4. Conclusion

The concentration distribution behavior of contaminants is depicted by an analytical solution of homogeneous finite porous medium with point source concentration that varies periodically with time against the groundwater flow under realistic boundary conditions and periodic velocity. Initially domain is not solute free. Pulse type periodic boundary conditions are assumed at the extreme end of the boundary. The obtained results may help to know the position and the time period of harmless concentration levels of the contaminated domain. The analytical expressions obtained here are useful to the study of salinity intrusion in groundwater, helpful in making quantitative predictions on the possible contamination of groundwater supplies resulting from groundwater movement through buried wastes. The obtained solution is based on simplified porous geometry and properties, such as constant dispersion, homogeneity, non-reactive pollutants etc. These assumptions allowed us to get analytical solution, which is highly efficient. When any of the above porous simplifications becomes unacceptable, numerical methods, such as the finite element method or the boundary element method may apply to get the solution.

Appendix: Derivation of Laplace Inversion Transform

The procedure used to invert the Laplace transform is to evaluate the closed contour and used the residue theorem. Branch point must be excluded from inside the contour and the original solution. $c(x, T)$, can be obtained by finding the solution to

$$c(x, T) = \frac{1}{2\pi i} \int_{\Gamma-i\infty}^{\Gamma+i\infty} \bar{K}(x, p) \exp(pT) dp \quad (\text{A-1})$$

or

$$c(x, T) = \sum_i \text{Res}(i) + \frac{1}{2\pi i} \int_{\Omega} \bar{K}(x, p) \exp(pT) dp \quad (\text{A-2})$$

where $\text{Res}(i)$ are the residue at the poles which lie to the left of the line $p = \Gamma$ and outside of the contour Ω . equation (18) is,

$$\begin{aligned} \bar{K}(x, p) = & \left[C_0 \exp(-\beta L) \left\{ \frac{1}{(p-\gamma^2)} \{ 2 - (2 - mT_0) \exp(-(p-\gamma^2)T_0) \} \right. \right. \\ & - \frac{m}{(p-\gamma^2)^2} \{ 1 - \exp(-(p-\gamma^2)T_0) \} \left. \left. \frac{\cosh(Mx-ML_1)}{\cosh(LM-L_1M)} \right\} \right. \\ & \left. - \frac{C_i}{\left\{ p - \frac{D_0}{R}(\alpha-\beta)^2 \right\}} \left\{ \frac{\exp\{(\alpha-\beta)L\} \cosh(Mx-ML_1)}{\cosh(LM-L_1M)} - \frac{(\alpha-\beta) \exp\{(\alpha-\beta)L_1\} \sinh(Mx-ML)}{M \cosh(LM-L_1M)} \right\} \right. \\ & \left. + \frac{C_i \exp(\alpha-\beta)x}{\left\{ p - \frac{D_0}{R}(\alpha-\beta)^2 \right\}} \right], \text{ where } M = \sqrt{\frac{Rp}{D_0}} \end{aligned}$$

(A-3)

Poles of the expressions are $p = \gamma^2$, $p = \gamma^2$ pole of order 2, $p = -\frac{(n+1/2)^2 D_0}{R(L-L_1)^2}$, $p = \frac{D_0}{R}(\alpha-\beta)^2$, $p = -\frac{(n+1/2)^2 D_0}{R(L-L_1)^2}$, respectively.

Therefore the residues of expressions with these poles may get as follows:

$$Res \text{ (at } p = \gamma^2) = \lim_{p \rightarrow \gamma^2} (p - \gamma^2) f(p) \exp(pT)$$

(A-4)

$$Res \text{ (at } p = \gamma^2 \text{ of order 2)} = \lim_{p \rightarrow \gamma^2} \frac{1}{1!} \frac{d}{dp} \{ (p - \gamma^2)^2 f(p) \exp(pT) \}$$

(A-5)

$$Res \text{ (at } p = -\frac{(n+1/2)^2 D_0}{R(L-L_1)^2}) = \lim_{p \rightarrow -\frac{(n+1/2)^2 D_0}{R(L-L_1)^2}} f(p) \exp(pT)$$

(A-6)

$$Res \text{ (at } p = \frac{D_0}{R}(\alpha-\beta)^2) = \lim_{p \rightarrow \frac{D_0}{R}(\alpha-\beta)^2} \left(p - \frac{D_0}{R}(\alpha-\beta)^2 \right) f(p) \exp(pT)$$

(A-7)

$$Res \text{ (at } p = -\frac{(n+1/2)^2 D_0}{R(L-L_1)^2}) = \lim_{p \rightarrow -\frac{(n+1/2)^2 D_0}{R(L-L_1)^2}} f(p) \exp(pT)$$

(A-8)

where $f(p)$ is the corresponding expression of the pole.

Finally sum of the all residues as,

$$\begin{aligned} & L^{-1} \left\{ \frac{\{ 2 - (2 - mT_0) \exp(-(p-\gamma^2)T_0) \} \cosh(Mx-ML_1)}{(p-\gamma^2) \cosh(LM-L_1M)} \right\} - L^{-1} \left\{ \frac{m \{ 1 - \exp(-(p-\gamma^2)T_0) \} \cosh(Mx-ML_1)}{(p-\gamma^2)^2 \cosh(LM-L_1M)} \right\} \\ & - L^{-1} \left\{ \frac{\exp\{(\alpha-\beta)L\} \cosh(Mx-ML_1)}{\left\{ p - \frac{D_0}{R}(\alpha-\beta)^2 \right\} \cosh(LM-L_1M)} \right\} + L^{-1} \left\{ \frac{(\alpha-\beta) \exp\{(\alpha-\beta)L_1\} \sinh(Mx-ML)}{\left\{ p - \frac{D_0}{R}(\alpha-\beta)^2 \right\} \cosh(LM-L_1M)} \right\} \\ & + L^{-1} \left\{ \frac{\exp(\alpha-\beta)x}{\left\{ p - \frac{D_0}{R}(\alpha-\beta)^2 \right\}} \right\} = \sum \text{Residues} \end{aligned}$$

(A-9)

The solution of advection-dispersion solute transport for periodic input condition in terms of $c(x, T)$ is defined in Eqs. (19a,b).

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5. References

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